A PRIMER ON THE TAGUCHI METHOD

SECOND EDITION

Ranjit K. Roy

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Preface

My exposure to the Taguchi methods began in the early 1980s when I was employed with General Motors Corporation at its Technical Center in Warren, Mich. At that time, manufacturing industries as a whole in the Western world, in particular the automotive industry, were starving for practical techniques to improve quality and reliability. Having perfected his concepts in Japan and a few places elsewhere during the late 1940s, Dr. Taguchi introduced his quality improvement methods to the United States in the early 80s. His focus was to optimize performance and make design robust by use of the statistical technique known as design of experiments (DOE), which was originally introduced by R.A. Fisher in England in the 1920s.

To make the technique more effective and easy to use, Dr. Taguchi recommended a standardized version of DOE and devised ways for practical application and analysis of results. For quality improvement champions, this was an attractive tool. Manufacturing organizations of all kinds readily learned and applied the methods to benefit the design of numerous products and processes. Remarkable progress was made by many throughout the 1980s. For automotive manufacturers, the process of catching up in quality with foreign manufacturers was launched.

Those of us who were actively promoting the quality improvement effort within our organizations and had the responsibilities to implement the technique were challenged to find better ways to teach and apply the technique. In the early 80s, there were only a handful of books and hardly any documented application examples. The primary reference in the market was Dr. Taguchi's own book, *System of Experimental Design* (Quality Resources, 1987). That book and related training materials were my first resources to explain the Taguchi technique to others in a simpler form. A credit to the effectiveness and acceptance of the Taguchi technique is that there now are more than a dozen textbooks and thousands of published reports available to practitioners.

Following the overwhelming surge of interest in learning and implementing the technique by manufacturing companies of all kinds, the focus shifted in the mid 1990s. Introduction of general and company-wide quality improvement disciplines such as ISO/QS-9000 and Six Sigma unintentionally diminished the priority and funding for activities that required special techniques. These general quality systems, which were value-added and beneficial to most businesses, also required use of statistical techniques like DOE, Taguchi methods, statistical process control (SPC), and so on, but for most companies these were among the last few things to do and easily postponed.

The turn of the 21st century saw the beginning of a downturn in the economy. Auto companies were downsizing; most didn't have much money to spend on training as business survival overshadowed quality improvement concerns. Now as the economy is starting to show signs of recovery, manufacturing companies can refocus on implementing statistical techniques.

While teaching, training, and practicing in the 1990s, I found greater demand for a reference on the application of the technique rather than on the theory. Thus, the first edition of *A Primer on the Taguchi Method* (Society of Manufacturing Engineers, 1990) introduced basic concepts through application examples and case studies. This led to my second book, *Design of Experiments Using the Taguchi Approach* (John Wiley & Sons, 2001), which covered a minimal amount of theory while describing many more applications in detail. Both books were received favorably by readers in academia, business, and industry.

HIGHLIGHTS OF THE SECOND EDITION

The following are highlights of additions and changes in this second edition of the *Primer*:

- Chapters 1–4: Minor changes in clarity and in the definitions of quality
- Chapter 5: Taguchi *robust design* strategy and the *two-step optimization* technique added
- Chapter 6: Advanced analysis of multiple-sample results described
- Chapter 7: Relationship of the loss function to other performance and capability statistics defined
- Chapter 8: Comprehensive experiment planning discussions included, and transformation to overall evaluation criteria revised and expanded
- Chapter 9: An example depicting current practices in application to production problem solving added

ACKNOWLEDGMENTS

This book would not be in circulation today without the Society of Manufacturing Engineers undertaking continued printing of the first edition after the original publisher folded technical publishing activities. For this second edition, I am indebted to Rosemary Csizmadia of SME, who was relentless in her conviction of market demand for the book even in difficult economic times. I would also like to thank Ellen Kehoe of SME for her grasp of language and insight into the technology in editing the manuscript.

I am grateful to my professional and business associates, Larry Smith, Mike Comerford, Greg Adams, Larry Tracey, Dave White, and Jay Chandra, for their trust in my ability to support their clients seeking training and application of the Taguchi technique. I would like to express my sincere thanks to Larry Smith, in particular, who spent an extraordinary amount of time on a meticulous review, and to Andrea Stamps, Kush Shah, Side Zhao, Fred Schenkelberg, and Pradeep Kumar for their detailed and constructive suggestions.

Finally, I thank my wife, Krishna, who has put up with my dedication to the life of an independent consultant, trainer, and author for more than two decades.

Ranjit K. Roy
January 2010

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1 Quality Through Product and Process Optimization

BACKGROUND

Mankind has always had a fascination with quality. Today's technology is testimony to man's incessant desire to provide a higher level of quality in products and services to increase market share and profits. Sometimes quality is essential. A pacemaker that controls heart action must operate continuously and precisely. An erratic pacemaker is valueless, useless, and dangerous.

Driven by the need to compete on price and performance and to maintain profitability, quality-conscious manufacturers are increasingly aware of the need to optimize products and processes. Quality achieved by means of design optimization is found by many manufacturers to be cost effective in gaining and maintaining a competitive position in the world market.

DESIGN OF EXPERIMENTS—THE CONVENTIONAL APPROACH

The technique of defining and investigating all possible conditions in an experiment involving multiple factors is known as the *design of experiments* (DOE). In the literature, this technique is also referred to as *factorial design*. Design of experiments concepts have been in use since Sir Ronald A. Fisher's work in agricultural experimentation during the late 1920s. Fisher [1] successfully designed experiments to determine optimum treatments of land for agriculture to achieve maximum yield. Numerous applications of this approach, especially in the chemical and pharmaceutical industries, are cited in the literature. A thorough coverage of this subject is beyond the scope of this study, but the method and its advantages and disadvantages from an engineering point of view are illustrated by a simple example.

Consider a snack food company planning to introduce a new chocolate chip cookie in the market. The product designers have standardized all other ingredients except the amount of sugar and chocolate chips. Two levels of chips, C_1 and C_2, and two levels of sugar, S_1 and S_2, were selected (subscripts 1 and 2, respectively, refer to the low and high levels of each factor). To select the best combination of these ingredients that appeal most to potential customers, the market research group decided to conduct a survey of customer preference.

This is one of the simplest cases of design of experiments. It involves two factors (chips and sugar) at two different levels (high and low) that affect the taste of cookies. Such an experiment is described as a 2 × 2 factorial experiment. There are four (2^2) possible treatments or combinations. The responses to these factors, as obtained by a taste test, are given in Table 1-1.

Examination of customer response shows a 10% (55 – 45) increase in preference for sugar level S_2 at the low level (C_1) of chocolate chips, but the response increases to 15% (80 – 65) when more chips (C_2) are used. These increases are called the simple effect of sugar. On the other hand, for the higher amount of chips, the taste preference increased from 45% to 65% at sugar level S_1 and further increased to 80% with the higher sugar level S_2.

Table 1-1. Taste preference survey (in percent)

<table>
<tr><th rowspan="2">CHOCOLATE CHIPS LEVEL</th><th colspan="2">SUGAR LEVEL</th><th rowspan="2">Mean</th><th rowspan="2">Mean response (chocolate chips) ($C_2 - C_1$)</th></tr>
<tr><th>S_1</th><th>S_2</th></tr>
<tr><td>C_1</td><td>45</td><td>55</td><td>50.0</td><td rowspan="2">22.5</td></tr>
<tr><td>C_2</td><td>65</td><td>80</td><td>72.5</td></tr>
<tr><td>Mean</td><td>55.0</td><td>67.5</td><td>Grand mean 61.25</td><td></td></tr>
<tr><td>Mean response (sugar) ($S_2 - S_1$)</td><td colspan="2">12.5</td><td></td><td></td></tr>
</table>

The mean response, that is, the difference between the average effects at two levels of sugar (12.5%), is called the main effect of sugar. Similarly, the main effect for chocolate chips is 22.5%. It is important to note that, in this example, only the main effects are analyzed; no attempt is made to analyze the interactions between the factors. Interactions may or may not be present. The relative influence of the factors and interactions between various factors included in the study can be quantitatively determined by using the analysis of variance (ANOVA). This procedure is described in Chapter 6.

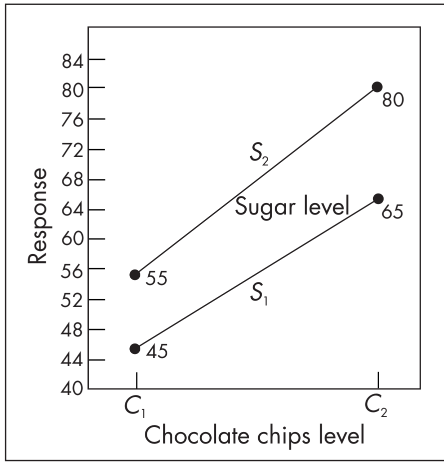


Figure 1-1. Factor effects

For the present, the degree of interactions for a 2 × 2 experiment can be determined from Figure 1-1, which graphs the response against one factor (C) for two levels of the second factor (S). Because the lines for the two levels, S_1 and S_2, are almost parallel, the factors (S and C) are said to be independent, and little or no interaction is assumed to exist. Nonparallel lines would indicate the presence of some interaction. Highly skewed lines or a higher angle between lines (need not be intersecting) would indicate strong interaction between the two factors. Figure 1-1 indicates only a slight interaction between the two factors (sugar and chocolate chips).

In the above example there were only two factors, each at two different levels. It would be rather easy to manufacture four types of cookies reflecting all possible combinations of the factors under study and to subject them to a market survey.

For a full factorial design, the number of possible designs, N, is

$$N = L^m \tag{1.1}$$

where L = number of levels for each factor and m = number of factors.

Thus, if the qualities of a given product depend on three factors, A, B, and C, and each factor is to be tested at two levels, then Eq. (1-1) would result in 2^3 (8) possible design configurations. This three-factor, two-level experiment is represented by Table 1-2.

In this configuration of factors and levels, the test matrix is still easily managed, and every combination can be investigated. Each box in the table is called a "cell." To improve accuracy, several observations are made per cell, and the significance of the factors' influences on the variability of results is determined by statistical analysis (ANOVA).

Now consider the case where the cookies under consideration have 15 different ingredients at two levels each. In this case, 2^{15} (32,768) possible varieties of cookies need to be investigated before the most desirable recipe can be established. A market research program of this magnitude would be exorbitant in cost and time. Techniques such as fractional (or partial) factorial experiments are used to simplify the experiment. Fractional factorial experiments investigate only a fraction of all possible combinations. This approach saves considerable time and money but requires rigorous mathematical treatment, both in the design of the experiment and in the analysis of the results. Each experimenter may design a different set of fractional factorial experiments.

So, while factorial and fractional factorial designs of experiments are widely and effectively used, they suffer from the following limitations:

1. The experiments become unwieldy in cost and time when the number of variables is large.
2. Two designs for the same experiment may yield different results.
3. The interpretation of the experimental results with a larger number of factors may be difficult due to lack of clear design and analysis guidelines.

In this part of the science of designing experiments, Dr. Genichi Taguchi of Japan proposed an innovative method. He simplified and standardized fractional factorial designs in such a manner that

Table 1-2. Test matrix with three factors at two levels

	A_1		A_2		Average
	B_1	B_2	B_1	B_2	
C_1					
C_2		"cell"			
Average					

two engineers conducting tests thousands of miles apart would use designs of similar size and expect to obtain consistent results.

Taguchi contributed discipline and structure to the design of experiments. The result is a standardized design methodology that can easily be applied by investigators. Furthermore, designs for the same experiment by two different investigators will yield similar data and will lead to similar conclusions. Taguchi overcame the limitations of factorial and fractional factorial experiments.

DESIGN OF EXPERIMENTS—THE TAGUCHI APPROACH

To make the DOE easier and more attractive to industrial practitioners, Dr. Taguchi proposed the following considerations for application of the technique:

1. *Definition of quality* – Taguchi defined quality in terms of minimum loss to society (described in detail in Chapter 2), which in measurable engineering terms translates into consistency of performance. Regardless of application, whether it is a product or a process, or how the results are measured, consistency in performance is considered as a primary attribute. Consistency is achieved when performance is close to the target with least variation. To improve quality, Taguchi proposed a two-step optimization approach:
    a. Find the factor-level combination that reduces performance variability.
    b. Adjust the factor levels that bring performance closer to the target.
2. *Standardized DOE* – For designing experiments, Taguchi utilized a special set of tables, called orthogonal arrays

(OAs), which represent the smallest fractional factorials and are used for most common experiment designs.

3. *Robust design strategy* – To make products and processes insensitive to the influence of uncontrollable (noise) factors, Taguchi incorporates a formal way to include noise factors in the experiment layout. This new structure (called outer array design) facilitates the use of experiments of smaller size to study the effects of a larger number of noise factors, which leads to a favorable performance with the mean close to the target and reduced variation around the mean.
4. *Loss function* – The mathematical formula associated with the concept of the loss function proposed by Taguchi allows a simple way to quantify the improvements in monetary units. The concepts can be easily used to express predicted improvement from DOE results in terms of expected cost savings.
5. *Signal-to-noise (S/N) analysis* – For analysis of results from multiple-sample tests, use of signal-to-noise ratios instead of the results makes the analysis of DOE results much easier. In addition, the logarithmic transformation of the results in terms of S/N ratios empowers the prediction of improvement in performance from the analysis.

EXERCISES

1-1. What are the three main disadvantages of the conventional design of experiments approach as compared with Taguchi's method?

1-2. Which one of the two factor effect graphs in Figure 1-2 indicates the existence of an interaction between the two factors of an experiment?

1-3. A product involves three primary parameters at three different levels of each. To optimize the product, a full factorial design is planned for experimental evaluations. How many possible design configurations need to be tested to achieve the objective?

1-4. Draw a factor graph for the experiment shown in Table 1-3 and discuss the results.

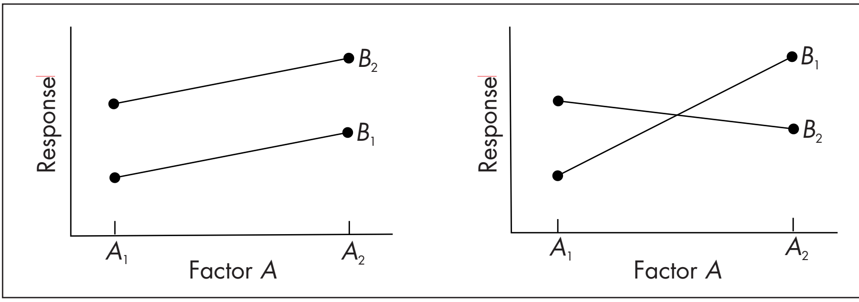


Figure 1-2. Factor effect graphs

Table 1-3. Two-factor experiment data

	A			
B	A_1	A_2	Average	Response to B ($B_2 - B_1$)
B_1	40	70	B_1 =	
B_2	65	45	B_2 =	
Average	A_1 =	A_2 =		
Response to A ($A_2 - A_1$)				

2 Taguchi Approach to Quality and Cost Improvement

BACKGROUND

After the Second World War, Allied forces found the quality of the Japanese telephone system to be extremely poor and totally unsuitable for long-term communication purposes. To improve the system to a state-of-the-art level, the Allied command recommended that Japan establish research facilities similar to the Bell Laboratories in the United States. The Japanese founded the Electrical Communication Laboratories (ECL), with Dr. Genichi Taguchi in charge of improving R&D productivity and enhancing product quality. Taguchi observed that a great deal of time and money was expended in engineering experimentation and testing, with little emphasis on the process of creative brainstorming to minimize the expenditure of resources.

Taguchi started to develop new methods to optimize the process of engineering experimentation. He developed the techniques that are now known as the Taguchi Methods. His greatest contribution lies not in the mathematical formulation of the design of experiments (DOE) but rather in the accompanying philosophy. His approach is more than a method to lay out experiments. It is a concept that has produced a unique and powerful quality improvement discipline that differs from traditional practices.

Two completely opposing points of view are commonly held about Taguchi's contribution to the statistical design of experiments. One view holds that his contribution to the field of quality control is one of the most significant developments of the last few decades. The other view maintains that many of the ideas proposed in Taguchi's approach are neither new nor were they developed by him. This text will not resolve this controversy but will explain application principles and document successful case studies using Taguchi's methods. These new techniques were transplanted to the United States in the early 1980s and created significant changes in quality engineering methods in this country. The Taguchi approach has been successfully applied in several industrial organizations and has completely changed their outlook on quality improvement activities.

TAGUCHI PHILOSOPHY

Taguchi espoused an excellent philosophy for quality control in the manufacturing industries. Indeed, his doctrine is creating an entirely different breed of engineers who think, breathe, and live quality. He has, in fact, given birth to a new quality culture in this country. Ford Motor Company, for example, decreed in the early 1990s that all Ford Motor and suppliers' engineers be trained in the Taguchi methodology and that these principles be used to resolve quality issues. Taguchi's philosophy has far-reaching consequences, yet it is founded on three very simple and fundamental concepts. The whole of the technology and techniques arise entirely out of these three ideas. These concepts are:

1. Quality should be designed into the product and not inspected into it.
2. Quality is best achieved by minimizing the deviation from a target. The product should be so designed that it is immune to uncontrollable environmental factors.
3. The cost of quality should be measured as a function of deviation from the standard, and the losses should be measured system-wide.

Taguchi built on W.E. Deming's observation that 85% of poor quality is attributable to the manufacturing process and only 15% to the worker. Hence, Taguchi developed manufacturing systems that were "robust" or insensitive to daily and seasonal variations of environment, machine wear, and other external factors. The three principles were his guides in developing these systems, testing the factors affecting quality production, and specifying product parameters.

Taguchi believed that the better way to improve quality was to design and build it into the product. Quality improvement starts at the very beginning, that is, during the design stages of a product or a process, and continues through the production phase. He proposed an “off-line” strategy for developing quality improvement early in the design phases in place of an attempt to inspect quality into a product on the production line. Taguchi observed that poor quality cannot be improved by the process of inspection, screening, or salvaging. No amount of inspection can put quality back into the product; inspection merely treats a symptom. Therefore, quality concepts should be based on, and developed around, the philosophy of prevention. The product design must be so robust that it is immune to the influence of uncontrolled application and environmental factors on the manufacturing processes. Taguchi was insistent on addressing quality up-front in design for much higher return on investment.

Taguchi’s second concept deals with actual methods of improving the quality of products. He contended that quality is directly related to the deviation of a design parameter from the target value, not to conformance to some fixed specifications. A product may be produced with properties skewed toward one end of an acceptance range yet show shorter life expectancy. However, by specifying a target value for the critical property and developing manufacturing processes to meet the target value with little deviation, the life expectancy may be much improved.

Taguchi’s third concept calls for measuring deviations from a given design parameter in terms of the overall life cycle costs of the product. These costs would include the cost of scrap, rework, inspection, returns, warranty service calls, and/or product replacement. These costs provide guidance regarding the major parameters to be controlled.

Taguchi views quality improvement as an ongoing effort. He continually strives to reduce variation around the target value. A product under investigation may exhibit a distribution that has a mean value different from the target value. The first step toward improving quality is to achieve the population distribution as close to the target value as possible. To accomplish this, Taguchi designs experiments using specially constructed tables known as orthogonal arrays (OAs). The use of these tables makes the design of experiments very easy.

A second objective of manufacturing products to conform to an ideal value is to reduce the variation or scatter around the target. To accomplish this objective, Taguchi cleverly incorporates a unique way to treat noise factors. Noise factors, according to his terminology, are factors that influence the response of a process but cannot be economically controlled. Noise factors such as weather conditions, machinery wear, and so on, are usually the prime sources for variations. Through the use of what he calls the outer arrays, Taguchi devised an effective way to study their influence with the least number of repetitions. The end result is a "robust" design affected minimally by noise, that is, with a high signal-to-noise (S/N) value.

To achieve desirable product quality by design, Taguchi recommends a three-stage process, as follows:

1. System design
2. Parameter design
3. Tolerance design

The focus of the *system design* phase is on determining the suitable working levels of design factors. It includes designing and testing a system based on the engineer's judgment of selected materials, parts, and nominal product/process parameters based on current technology. Most often it involves innovation and knowledge from the applicable fields of science and technology.

While system design helps to identify the working levels of the design factors, *parameter design* seeks to determine the factor levels that produce the best performance of the product/process under study. The optimum condition is selected so that the influence of the uncontrolled factors (noise factors) causes minimum variation of system performance. This text deals solely with parameter design.

Tolerance design is a step used to fine-tune the results of parameter design by tightening the tolerance of factors with significant influence on the product. Such steps will normally lead to identifying the need for better materials, buying newer equipment, spending more money for inspection, and so on.

Detailed discussion of system design and tolerance design is beyond the scope of this text.

CONCEPT OF THE LOSS FUNCTION

The concept of the "total loss function" employed by Dr. Taguchi has forced engineers and cost accountants to take a serious look at the quality control practices of the past. The concept is simple but effective. Taguchi defines quality as "the total loss imparted to society from the time a product is shipped to the customer." The loss is measured in monetary terms and includes all costs in excess of the cost of a perfect product. The definition can be expanded to include the development and manufacturing phases of a product.

A poorly conceived and designed product begins to impart losses to society from the embryonic stage and continues to do so until steps are taken to improve its functional performance. There are two major categories of loss to society with respect to the product quality. The first category relates to losses incurred as a result of harmful effects to society (for example, pollution), and the second category relates to losses arising because of excessive variation in functional performance. In this book, the total loss function refers essentially to the second category.

The conventional method of computing the cost of quality is based on the number of parts rejected and reworked in a production environment. This method of quality evaluation is incapable of distinguishing between two samples, both within the specification limits but with different distributions of targeted properties. Figure 2-1 shows the conventional method and Taguchi's view of the loss function. This graph depicts the loss function as a function of deviation from an ideal or the target value of a given design parameter. Here T represents the target value or the most desirable value of the parameter under consideration. This parameter may be a critical dimension, color of the product, surface finish, or any other characteristic that contributes to the customer's

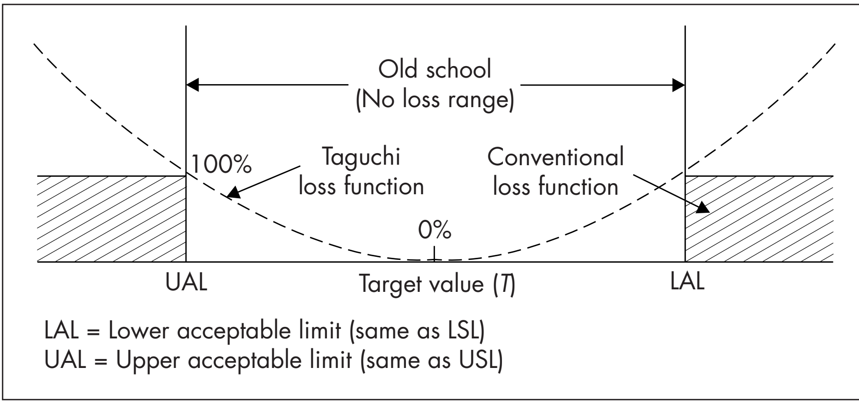


Figure 2-1. Taguchi and conventional loss functions

conception of quality. How this ideal value of the parameter was arrived at and how significant this value is in achieving quality goals will be evident later.

UAL and LAL in Figure 2-1 represent upper and lower acceptable limits of a design parameter, respectively. Normally the product is functionally acceptable if the value of the specified parameter is within the range between the UAL and LAL limits. No societal loss is assumed to occur; the product is shipped to the consumer. However, outside these limits, as shown by the crosshatched region, 100% functional deterioration occurs, and the product is either discarded, reworked, or subjected to salvage operations. Every attempt is made to control the manufacturing process to maintain the product within the acceptable limits.

However, according to Taguchi, there is no sharp cutoff in the real world on situations just before and beyond the LAL and UAL points. Typically, performance begins to gradually deteriorate as the design parameter deviates from its optimum value. Therefore, Taguchi proposed that the loss function be measured by the deviation from the ideal value. This function is continuous, as shown by the dotted line in Figure 2-1. Product performance begins to suffer when the design parameters deviate from the ideal or the target value. Taguchi's definition clearly puts more emphasis on customer satisfaction, whereas previously all definitions were concerned with the producer. Optimum customer satisfaction can be achieved by developing the products that meet the target value on a consistent basis. It may be worthwhile to mention that Taguchi allows for more than 100% loss imparted by a product. Such cases can occur when a subsystem results in a failure of the entire system or when a system fails catastrophically. Thus, the single most important aspect of Taguchi's quality control philosophy is the minimization of variation around the target value.

A case study conducted by the Sony Corporation makes it abundantly clear that these two schools of thought are significantly different from each other and indeed affect customer satisfaction. In the early 1980s, Sony manufactured one of its color television sets in Japan as well as in the United States. The TVs from both sources were intended for the U.S. market and had identical design and system tolerances. Yet American consumers consistently preferred the color characteristics of TV sets manufactured overseas.

A study was conducted to determine a cause for the difference in customer preference. The results indicated that the frequency distributions for the sets manufactured in United States and those manufactured in Japan were significantly different, as shown in Figure 2-2. Plants in both countries produced TVs with color density within the tolerance range. None or a limited number of televisions with out-of-tolerance color characteristics were shipped to the consumer. However, the U.S.-built sets followed a somewhat flat distribution consistent with a go/no-go philosophy, while the product manufactured in Japan followed a normal distribution with smaller deviation from the target value. The large scatter, observed in the performance characteristics of the product manufactured in the U.S., as is evident from Figure 2-2, was responsible for significantly lower customer preference for these sets. Once the process in the U.S. plant was improved, which led to the production of the frequency distribution similar to the TVs produced in Japan, customer satisfaction with the U.S. product achieved the level of satisfaction seen with the imported sets. The Sony case demonstrated that quality is more than just producing between upper and lower limits; quality is achieving the target as much as possible and limiting deviations from the target.

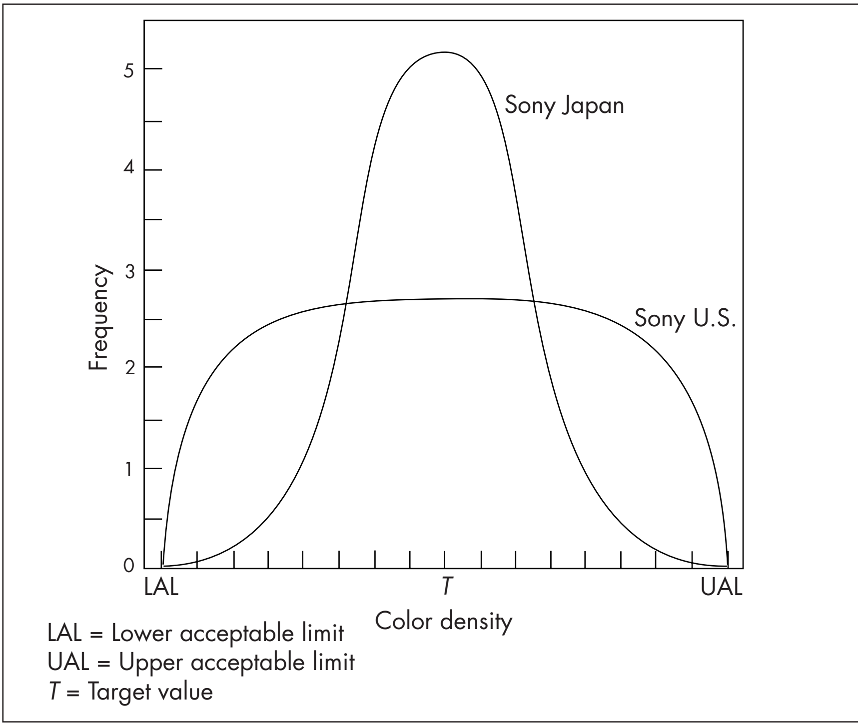


Figure 2-2. Color density distributions

Consider another example, which will further support this concept of quality. Two batches of main bearings for an internal combustion engine were received from two different sources, A and B, for a new engine development program. Under laboratory conditions, bearings from source B wore much faster than those from source A. To pinpoint the cause of the unequal wear, selected performance characteristics of the bearings were measured and posted. Both batches of bearings were within the design specifications. However, the source B bearings consistently measured a mean diameter on the larger side of the tolerance limits, as depicted in Figure 2-3. Although within the tolerance band, the larger diameter resulted in excessive clearance. Bearing analysis later revealed that excessive clearance adversely affected the oil film thickness, causing the poor wear properties of this batch.

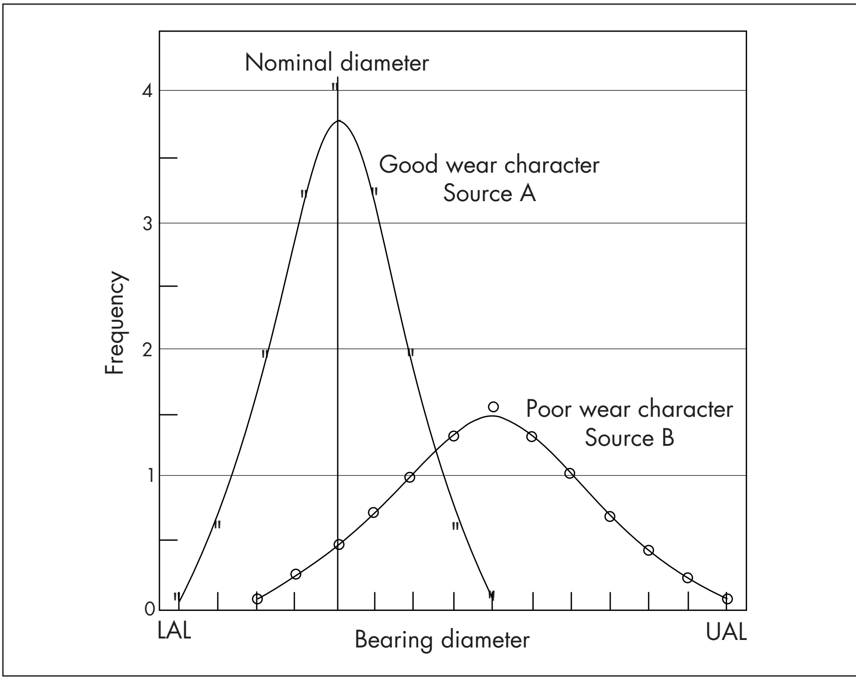


Figure 2-3. Bearing diameter distribution

The problem was solved by adjusting the manufacturing process to maintain bearing diameter near the target value.

The loss function and its implications are discussed in detail in later sections. At present, it is important to note that:

- The quality loss function is a continuous function and is a measure of deviation from the target value. The conformance to specification limits LAL and UAL is an inadequate measure to define the quality loss function.
- Quality loss is related to product performance characteristics and can best be minimized by designing quality into the product. Prevention of poor quality is less costly than rework and yields far better returns.
- Quality loss results from customer dissatisfaction and should be measured system-wide rather than at a discrete point in the manufacturing process.

- Quality loss is a financial and societal loss.
- Minimization of quality loss is the only way to be competitive and survive in today's competitive business environment.

EXPERIMENT DESIGN STRATEGY

Dr. Taguchi utilized a special set of orthogonal arrays (OAs) to lay out his experiments. The use of Latin squares orthogonal arrays for experiment designs dates back to the time of World War II. By combining the orthogonal Latin squares in a unique manner, Taguchi prepared a new set of standard OAs to be used for a number of experimental situations. A common OA for two-level factors is shown in Table 2-1. This array, designated by the symbol L_8 (or L-8), is used to design experiments involving up to seven two-level factors. The array has eight rows and seven columns. Each row represents a trial condition with factor levels indicated by the numbers in the row. The vertical columns correspond to the factors specified in the study.

The columns of all orthogonal arrays are balanced in two ways. First, the columns are balanced within themselves such that they all have an equal number of levels of the factor. Second, the columns are balanced between any two columns such that together they form an equal number of possible combinations. For example, each column in an L_8 array (Table 2-1) contains four one-level and

Table 2-1. Orthogonal array $L_8(2^7)$

TRIAL \ FACTOR	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

four two-level conditions for the factor assigned to the column. Two two-level factors combine in four possible ways, such as (1,1), (1,2), (2,1), and (2,2). When two columns of an array form these combinations the same number of times, the columns are said to be balanced or orthogonal. Note that any two columns of an L_8 (2^7) array have the same number of combinations of (1,1), (1,2), (2,1), and (2,2). Thus, all seven columns of an L are orthogonal to each other.

The OA facilitates the experiment design process. To design an experiment is to select the most suitable orthogonal array, assign the factors to the appropriate columns, and finally, describe the combinations of the individual experiments, called the trial conditions. Let us assume that there are at most seven two-level factors in the study. Call these factors A, B, C, D, E, F, and G and assign them to columns 1, 2, 3, 4, 5, 6, and 7, respectively, of L_8. The table identifies the eight trials needed to complete the experiment and the level of each factor for each trial run. The experiment descriptions are determined by reading the numerals 1 and 2 appearing in the rows of the trial runs. Obviously, when in use for experiment design, the numbers in the columns of the orthogonal array represent the level of the factors assigned to the column. A full factorial experiment would require 2^7 or 128 runs but may not provide appreciably more useful information.

The array forces all experimenters to design almost identical experiments. Experimenters may select different designations for the columns, but the eight trial runs will include all combinations independent of column definition. Thus, the OA assures consistency of design by different experimenters.

ANALYSIS OF RESULTS

In the Taguchi method, the results of the experiments are analyzed to achieve one or more of the following three objectives:

1. To determine the trend of influence of factors and interactions under study.
2. To identify the significant factors and their relative influences on the variability of results.

3. To establish the best or the optimum condition for a product or a process, along with
   - An estimate of contribution of individual factors.
   - A prediction of expected response under the optimum conditions.

The optimum condition is identified by studying the main effects of each of the factors. The process involves minor arithmetic manipulation of the numerical results for average effects of factor levels and usually can be done with the help of a simple calculator. The main effects indicate the general trend of the influence of the factors. Knowing the characteristic, that is, whether a higher or lower value produces the preferred result, the levels of the factors that are expected to produce the best results can be predicted.

The knowledge of the contribution of individual factors is a key to deciding the nature of the control to be established on a production process. The analysis of variance (ANOVA) is the statistical treatment most commonly applied to the results of the experiment to determine the relative percent influence of an individual factor and to separate the significant factors from the insignificant ones. Study of the ANOVA table for a given analysis helps determine which of the factors need control and which do not.

Once the optimum condition is determined and expected performance (predicted value) is estimated, it is usually a required and good practice to run a confirmation experiment. As additional information, performance at any of the full factorial conditions (128 for L-8 experiment) can also be calculated from the results of experiments conducted. It should be noted that the optimum condition may not necessarily be among the many experiments already carried out, as the OA represents only a small fraction of all the possibilities.

Taguchi suggests two different routes to carry out the complete analysis. First, the standard approach, where the result of a single run, or the average of repetitive runs, is processed through main effect and ANOVA analyses, as identified above. The second approach, which Taguchi strongly recommends for multiple runs, is to use the signal-to-noise (S/N) ratio for the same steps in the analysis. S/N analysis determines the most robust set of operating conditions from variations within the results.

AREAS OF APPLICATION

Analysis

In the design of engineering products and processes, analytical simulation plays an important role, transforming a concept into the final product design. The Taguchi approach can be utilized to arrive at the best parameters for the optimum design configuration with the least number of analytical investigations. Although there are several methods available for optimization, using such simulations when the factors are continuous, the Taguchi method is the method that treats factors at discrete levels. Frequently this approach significantly reduces computer time.

Test and Development

Testing with prototypes is an efficient way to see how the concepts work when they are put into a design. Because experimental hardware is costly, the need to accomplish the objectives with the least number of tests is a top priority. The Taguchi approach of laying out the experimental conditions with standardized orthogonal arrays significantly reduces the number of tests and the overall testing time.

Process Development

Manufacturing processes typically have a large number of factors that influence the final outcome. Identification of their individual contributions and their intricate interrelationships is essential in the development of such processes. The Taguchi concepts used in such projects have helped many U.S. and Japanese companies realize significant cost savings in recent times.

Validation Testing

For many products, proper validation testing requires assurance of performance under numerous application factors and durability life cycles. Many products also are designed to be robust against many known noise conditions. Use of the Taguchi approach to lay out structured test plans can potentially save costs for product assurance.

Marketing and Advertising

While as a general capability for studying multiple variables at a time the experimental design technique always had potential for benefiting advertising and marketing efforts, it was not until Web-based advertising became popular in the late 1990s that the benefits of the DOE technique, particularly the Taguchi approach to experimental design, was demonstrated by many large consumer product companies. Today, many Web-based advertising companies routinely use the Taguchi DOE technique to optimize advertisement design or traffic.

Problem Solving

Production and manufacturing problems related to variations, rework, and rejects are common in industry. While many such issues may be resolved by common problem-solving disciplines, some require special techniques. Fortunately, the solution often is obtainable by properly adjusting many influencing factors rather than searching for innovative means. The Taguchi DOE is a powerful technique to investigate such technical issues and determine data-driven permanent solutions.

THE NEW APPROACH—ITS APPEAL AND LIMITATIONS

The Appeal

- Up-front improvement of quality by design and process development.
- Measurement of quality in terms of deviation from the target (loss function).
- Problem solution by team approach and brainstorming.
- Consistency in experimental design and analysis.
- Reduction of time and cost of experiments.

- Design of robustness into product/process.
- Reduction of variation without removing its causes.
- Reduction of product warranty and service costs by addressing them with the loss function.

Taguchi's design methodology, the common features of which are listed above, has wide-ranging applications. Generally speaking, experimental design using OAs can be applied where there are a large number of design factors. Taguchi's OAs for the design of experiments, signal/noise analysis, and cost guidance based on the loss function have made his approach increasingly popular among practicing engineers. Taguchi's extension of loss beyond the production line has necessitated a team-based approach to the application of DOE techniques, and this approach has been found to be highly effective.

Limitations

The most severe limitation of the Taguchi method is the need for proactive thinking and working as a group to address the quality improvement issues early in the product/process development. The technique is most effective when applied before the design of the product/process system is released. After the design variables are determined and their nominal values are specified, experimental design may not be cost effective. Also, though the method has wide-ranging applications, there are situations in which classical techniques are better suited; for example, in simulation studies involving factors that vary in a continuous manner, such as the torsional strength of a shaft as a function of its diameter, the Taguchi method may not be the best choice.

EXERCISES

2-1. There are two types of losses that society incurs because of the poor quality of a product. What are these losses?

2-2. Explain why the old definition of cost of quality is inadequate.

2-3. What is the most important idea of Taguchi's concept of achieving higher product quality?

2-4. Name three stages in the process of achieving desirable quality by design.

2-5. List some areas in your field where the Taguchi approach can be used to improve a product or the efficiency of a manufacturing process.

2-6. A manufacturer of hi-fi speakers uses a gluing operation at the last stage in the manufacturing process. Recently, because of a change in the bonding agent, the quality of the bond has been observed to be below specifications. Some engineers maintain that the poor quality of the bond is attributable not to the change of glue but rather to the accompanying application temperature. A 2×2 factorial experiment including two different glues at two application temperatures is planned. List at least three noise factors that may influence the outcome of the test.

3 Measurement of Quality

THE QUALITY CHARACTERISTIC

Every product is designed to perform some intended function. Some measurable characteristic, generally referred to as the quality characteristic, is used to express how well a product performs the function. Consider a light bulb; its quality can be measured in terms of its hours of life. For a machine automatically producing 2.00 inch diameter shafts, the deviation from this target dimension may be a quality characteristic. In a majority of cases, the quality characteristic may be a single measurable quantity such as weight, length, hours, and so on. For some products, subjective measurements like "good," "bad," "low," and "high" may be used. In other instances, subjective and objective evaluations may be combined into an Overall Evaluation Criteria (OEC, Chapter 8).

No matter how the quality of the product is measured—by a single criterion or by a combination of multiple criteria, the measure will possess one of the following three characteristics that indicates the direction of desirability of results:

- bigger is better
- smaller is better
- nominal is best

Suppose that we are investigating a pump to determine the best design parameters that produce the maximum flow rate. In this case, the quality of the design may be judged by the flow rate, measured in units of cubic feet per minute, which therefore will be of the characteristic "bigger is better."

If, on the other hand, the purpose of the study is to determine the least noisy pump, the noise measured in units of, say, decibels, will be of the type described by "smaller is better." When the object or process under study has a target value, as for a battery of 9.0 volts or a process to machine a cylinder with a 3.00 inch inside diameter, the measure of quality will possess the "nominal is best" characteristic.

In general engineering practices, performance of a product or process is termed 'result' or 'response' and is expressed in terms of any suitable units of measurements. In scientific experimental studies involving DOE, the term *quality characteristic* (QC) is used, along with its two attributes: units of measure and the direction of desirability. For example, in a study to improve the power output of an internal combustion engine, the selected QC may be expressed as, QC: power generated (horsepower, bigger is better).

VARIATION AS A QUALITY YARDSTICK

Variation is the law of nature. In nature, no two objects are absolutely alike. They could be very similar, but hardly identical. No two people are exactly alike. No two apples have precisely the same weight. Mother Nature likes variety. Variations in nature are often obvious to the human eye.

Consider man/machine-made items. Superficially, parts look and function alike. However, when examined closely, manufactured products also exhibit variation, which, unlike in nature, may not be obvious to the human eye. Two ballpoint pens of the same brand do not write in the same way; two light bulbs do not last the same amount of time; two appliances do not function in exactly the same manner; two engines of similar specifications do not perform identically. This is because products made for the same purpose will show factor influences and perform differently.

Generally speaking, the quality characteristic of a product varies in two ways. First, it differs from another of the same kind, and second, it differs from the desired (target) value. Consider five 9-volt transistor batteries. When their voltages are measured accurately with a voltmeter, they may display a range of 8.90, 8.95, 8.99, 9.20, and 9.20 volts. All of the batteries may work well for radios with a range of acceptance of 8.5 to 9.5 volts, exceeding the variation in these batteries. But for a sophisticated instrument, only batteries that exhibit a voltage very close to the target value, say, 8.95 to 9.05 volts, will operate the instrument properly. Batteries with excessive deviations from the target value may produce unreliable readings or may even damage the instrument.

The first kind of variation can be displayed by comparing one item with another. The maximum voltage variation among the batteries is 0.3 (9.2 to 8.9) volts. Although all of the batteries are nominally rated at 9 volts, most of them will deviate from this value. The deviation from this target or nominal value constitutes the other type of variation. These variations are shown in Figure 3-1. In Figure 3-1(a), the average value of the parameter deviates from the target value; the range of value (variation) is also excessive. Figure 3-1(b) shows the average on-target, but the variation is still excessive. Figure 3-1(c) illustrates the desired characteristic—on target and with narrow variation.

COST OF VARIATION

Early in his research, Dr. Taguchi observed that unexpected variation was common to all manufacturing processes and that it was the primary cause for rejection of parts. Parts were rejected upon inspection when they did not conform to a predefined specification. Rejection increases the cost of production. Often, 100% inspection is excessively costly or impractical; thus, a defective part may reach a customer and lead to warranty costs and customer dissatisfaction. Taguchi held that variation is costly even beyond the immediate factory production cost and that excessive variation causes loss of quality. He contended that the cure for quality loss is reduction of variation. Thus, he recommended that effort should be directed toward minimizing variation, with less emphasis placed on production within fixed tolerance limits.

QUALITY AND VARIATION

Taguchi viewed variation as a lack of consistency in the product, giving rise to poor quality. With this view, he developed methodologies aimed at reducing both of the elements of variation—(a) deviation

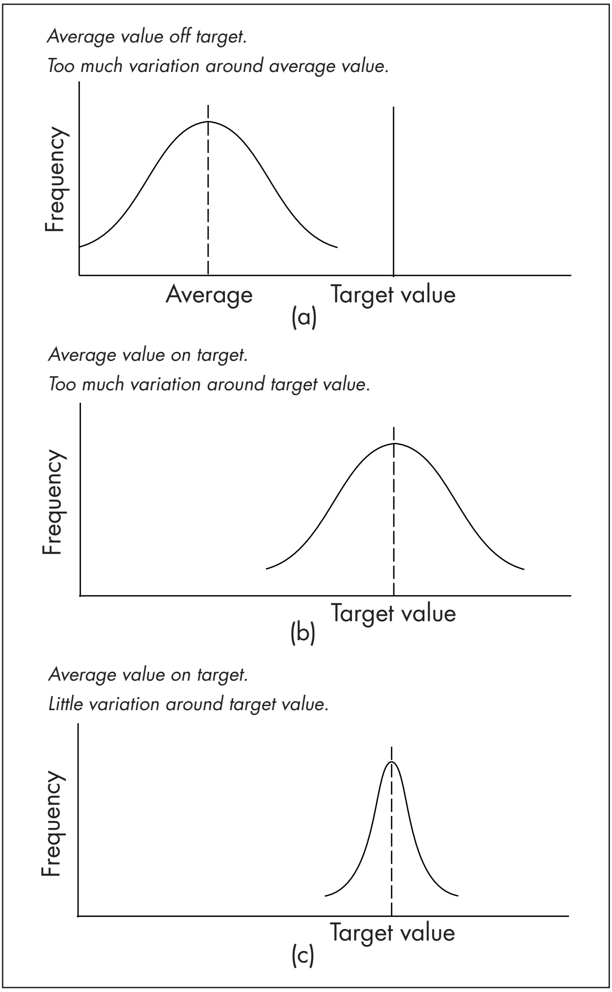


Figure 3-1. Typical quality distributions

from the target and (b) variation with respect to others in the group. In Figure 3-2, a typical quality measure of a product (similar to Bearing Dimension discussed in Chapter 2) is compared with the desired state. Note that the product mean value is off target and that the variation around the mean is large, though within upper and lower acceptance limits. A much narrower distribution

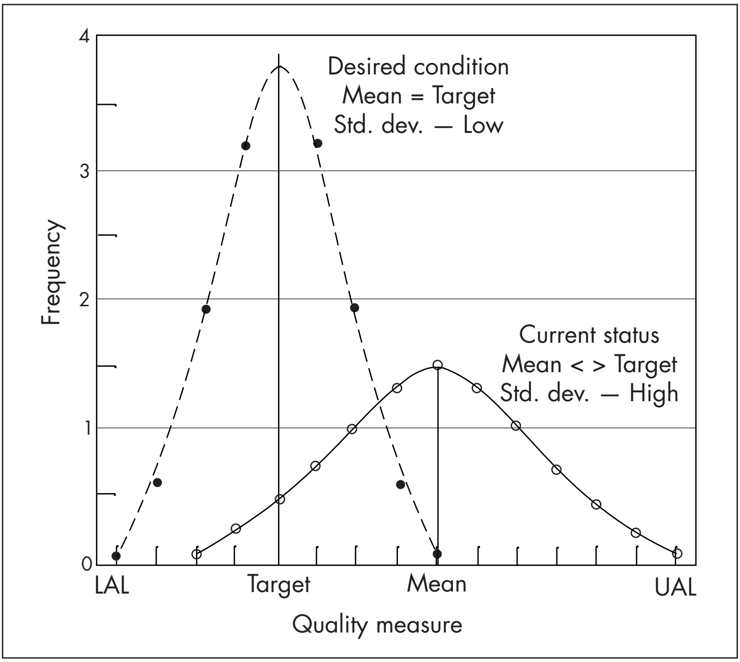


Figure 3-2. Representation of the Taguchi approach

is desired, with more frequent achievement of the target value and smaller variation around the target value.

How is this accomplished? What does it mean in terms of cost savings? The financial implications of variation will be covered in a later chapter. In this chapter, we will discuss in detail Taguchi's approach to variation reduction.

THE QUALITY WE ARE AFTER

The quality of a product or a process may be difficult to define in quantitative terms. Quality is what the customers perceive it to be; thus, quality varies from product to product and from customer to customer. The criteria customers use to judge the quality of a product are related to the satisfaction derived from the product and are numerous and often difficult to quantify. Research has shown that a lack of product consistency is a major factor in the perception of poor quality. Consistency (reduced level of variation) favorably affects most common elements of quality. To customers, quality may include service after delivery, ease of assembly, product performance, frequency of maintenance, and so on. Our focus is the element of quality reflected by the performance of a product or service. The Taguchi approach for reducing variation in performance is a two-step process:

1. Make the product/process perform in the best manner most of the time (less deviation from the target).
2. Make all products perform as identically as possible (less variation between the products).

TAGUCHI QUALITY STRATEGY

Taguchi's approach to enhance quality in the design phase involves two steps:

1. Optimizing the design of the product/process (system approach).
2. Making the design insensitive to the influence of uncontrollable factors (robustness).

When a product is optimum, it performs best under the available operating conditions. Depending on the specified performance, the optimum will imply that the product has achieved the most, the least, or the target value of the quality measure. Optimizing the design of a product means determining the right combination of ingredients or making the proper adjustments to the machine so that the best results are obtained.

Consider a baking process. Assume several bakers are given the same ingredients to bake a pound cake, the object being to produce the best-tasting cake. Within limits, they can adjust the amount of ingredients, but they can only use the ingredients provided. They are to make the best cake within available design parameters. Taguchi's approach would be to design an experiment considering all baking ingredients and other influencing factors, such as baking temperature, baking time, oven type (if a variable), and so on.

SELECTING DESIGN PARAMETERS FOR REDUCED VARIATION

In the last section, quality according to Taguchi's methodology was defined. Taguchi strives to attain quality by reducing the variation around the target. In an effort to reduce variations, he searched for techniques that allow variability to be reduced without necessarily eliminating the causes of variation. Often in an industrial setting, totally removing the causes of variation can be expensive. A no-cost or low-cost solution may be achieved by adjusting the levels and controlling the variation of other factors. This is what Taguchi tries to do through his *Parameter Design* approach. There is no cost or low cost in reducing variability in parameter design. Furthermore, the cost savings realized far exceed the cost of additional experiments needed to reduce variations.

The Taguchi method is most effective when applied to experiments with multiple factors. But the concept of selecting the proper levels of design factors, and reducing the variation of performance around the optimum/target value, can be easily illustrated through an example involving only one factor.

An electronic circuit that controlled the color characteristics of a television set was significantly influenced by the line voltage. The experimenter, wishing to select the right voltage, investigated the color quality at several input voltages. The influence of voltage variation on color quality is shown in Figure 3-3. If the desirable range of voltage for circuit design is between V_C and V_D, then what voltage should be specified for the circuit? Obviously, the choice should be a point within working voltage range V_C and V_D that provides stable color quality. In Taguchi terminology, one would look for the input voltage that reduces variation of the color quality. The experimenter would initially select a voltage at point B, so the variation around B, say, to B' or B'', would minimally affect the output, that is, the color quality. Voltage B is highly attractive because small fluctuations in the line voltage (B' to B'') will not significantly affect the color quality of the TV as perceived by the customer-user.

The objective for products involving multiple factors is similar but slightly more complex. The idea is to combine the factors at appropriate levels, each within the respective acceptable range,

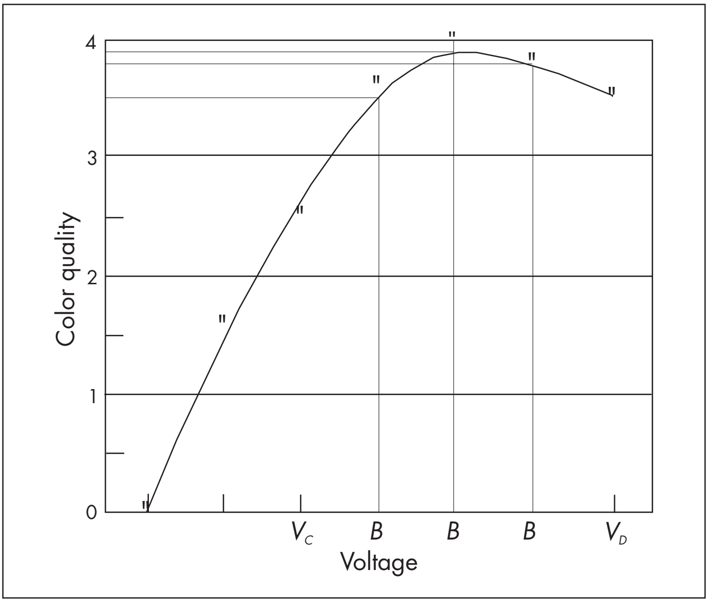


Figure 3-3. Color quality response curve

to produce the best result and yet exhibit minimum variation around the optimum result.

To see how the Taguchi technique is used for many factors, consider once again the process of determining the best recipe for a pound cake (Figure 3-4). The objective is to determine the right proportions of the five major ingredients—eggs, butter, milk, flour, and sugar, so that the recipe will produce the best cake most of the time (Figure 3-5). Based on the past experience of involved team members, the working ranges of these factors are established at the levels as shown in Figure 3-6. At this point, we face the following questions. How do we do determine the right combination? How many experiments do we need to run and in what combination?

COMMON TERMINOLOGY

The technique for laying out the conditions of experiments when multiple factors are involved has been known to statisticians for a long time. The technique was first introduced by

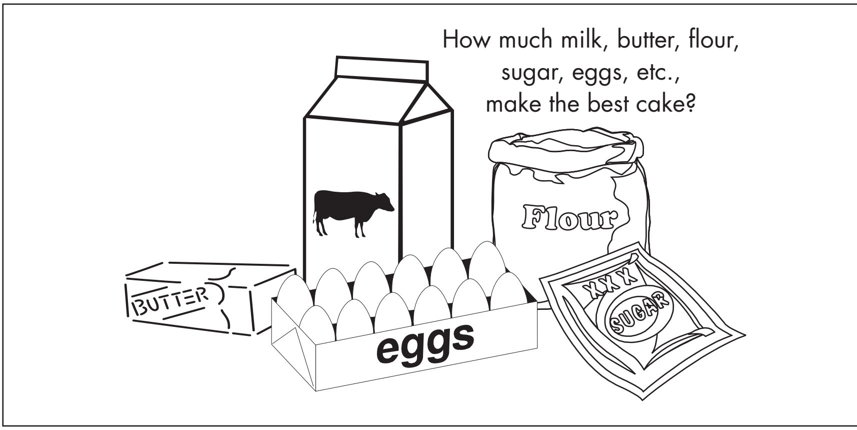


Figure 3-4. Cake baking experiment

Figure 3-5. Desirable result of an optimized baking process

Sir Ronald A. Fisher in England in the 1920s and is popularly known as the *factorial design of experiments*. The method helps an experimenter determine the possible combinations of factors and to identify the best combination. To determine the optimum combination, Dr. Taguchi prescribes carrying out a number of experiments under the conditions defined by the rules he has

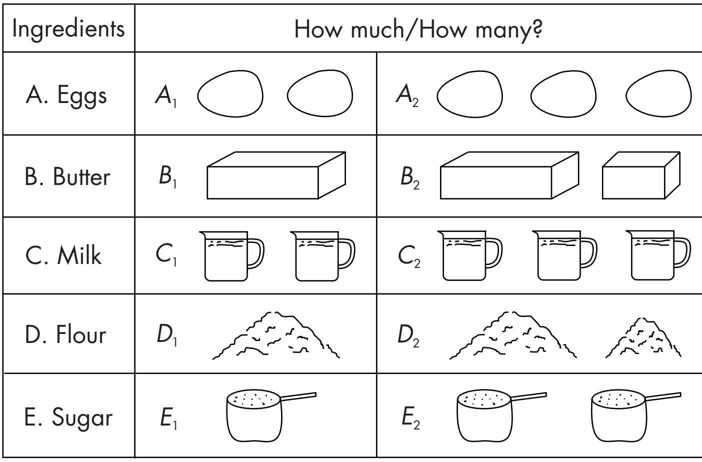


Figure 3-6. Factors and levels for a pound cake experiment

developed. In experimental layout, he uses the same principles as that of factorial design, except that his methods are much simplified and standardized.

In the cake example, with five factors each at two levels, there are 32 (2^5) combinations of all possible factors and levels. If we could bake 32 cakes, we would surely find the best-tasting one among these 32 cakes, and we would know the impact of each ingredient level on taste. For most industrial situations, carrying out a large number of experiments is not feasible.

Taguchi accomplishes the same objective with a smaller number of tests. He selects a particular eight-trial fractional factorial design (orthogonal array) that produces the most information regarding the best-tasting cake. The factor/level combinations for these eight experiments are defined by using the L_8 (2^7) orthogonal arrays. This orthogonal array (OA) is a table of eight rows and seven columns of numbers developed to design experiments with seven two-level factors. Orthogonal arrays are products of many years of statistical research that bears a high degree of confidence. They are so constructed such that the columns are balanced (equal number of levels) within each column, and also the columns are balanced (equal number of level combinations) between any two columns. Thus, an experiment planned using the balanced orthogonal arrays provides statistically meaningful results.

An OA experiment design leads to reduction of variation caused mainly by controllable factors. Uncontrollable factors (noise, dust, and so on) can be handled in two ways. First, the experiment trial can be repeated at different noise conditions. Second, the noise factors can be included in a second orthogonal array (called an outer array), which is used in conjunction with an inner array, the array of controllable factors.

Because OAs are used to define the unique experimental conditions as well as the noise factors, Taguchi calls the former design *inner array* and the latter *outer array.* When outer array experiments are performed, or when there are multiple samples tested in the individual experimental condition, the analysis involves transformation of the results into a *signal-to-noise ratio* (S/N). S/N follows a transformation of the trial results into a logarithmic scale, which changes the results of unknown nonlinear behavior into a linear relationship with the influencing factors. This process identifies the optimum condition and the expected performance with the least variability of the controllable as well as the uncontrollable factors.

The actual steps involved in designing the experiments using inner and outer arrays will be discussed in Chapter 5.

EXERCISES

3-1. How does Taguchi's view of quality differ from the conventional practice?

3-2. How does variation affect cost and quality?

3-3. What are the main causes of variation?

3-4. How is a product design optimized?

3-5. How does Taguchi make the design less sensitive to the noise factors?

3-6. What are orthogonal arrays?

3-7. What is implied by the term *parameter design* and what is its significance in achieving higher product quality?

4 Attractions and Benefits of the Taguchi Method

THE NEW DISCIPLINE

The Taguchi method offers two new powerful elements. First, the method is a disciplined way of developing a product or investigating complex problems. Second, it provides a means to cost-effectively investigate the available alternatives. Although Dr. Taguchi's method was built on well-developed concepts of optimization through the design of experiments, his philosophy regarding the value of quality and the procedure for carrying out experiments were new. The power and popularity of the method lies in the discipline rather than the technique itself. The attractiveness and the resultant potential for cost savings will be reviewed in this chapter.

The technique is applied in five steps, as follows:

1. Brainstorm the quality characteristics and design parameters important to the product/process under study.
2. Design the experiment and prescribe individual test recipes.
3. Conduct the experiments.
4. Analyze the results to determine the optimum conditions.
5. Run a confirmatory test(s) using the optimum conditions.

These steps are contrasted with typical current practice in Figure 4-1.

Brainstorming is a necessary and important step in the application process. The nature and content of the brainstorming is dependent on the type of project under study. Taguchi recommends the participation of all relevant functional organizations, including marketing. Suggested steps for brainstorming (experiment

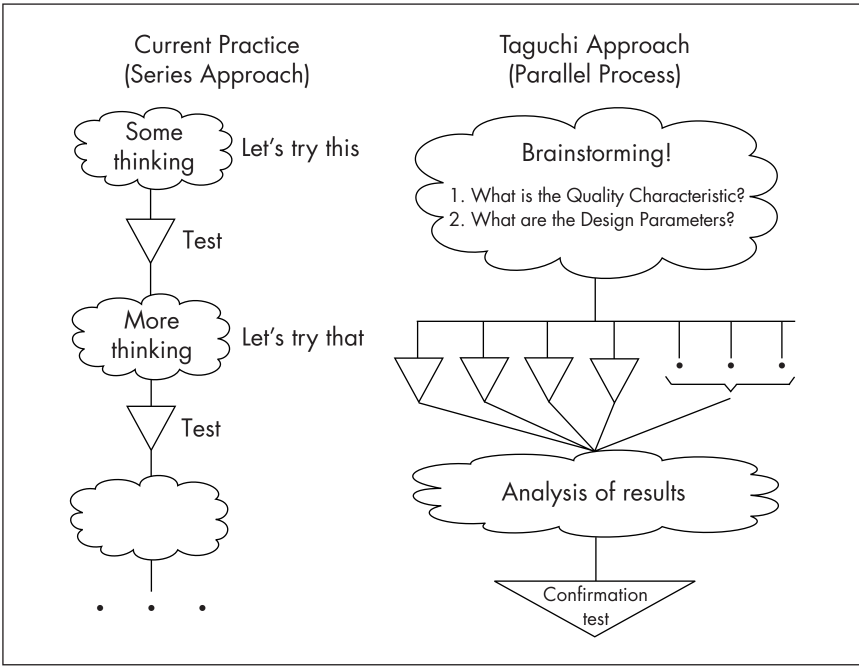


Figure 4-1. Comparison of current practice and the Taguchi approach

planning) for Taguchi experiment designs, along with some general guidelines, are described in Chapter 8.

Taguchi experiments are designed according to some strict rules. A set of orthogonal arrays (OAs) is used to design the experiments. A single OA may accommodate several experimental situations. Commonly used OAs are available for two-, three-, and four-level factors. Some standard arrays accommodate factors of mixed levels. In many situations, a standard OA is modified to suit a particular experiment that requires factors of mixed levels. The process of experiment design includes selecting the suitable OA, assigning the factors to the appropriate columns, and determining the conditions for the individual experiments. When noise factors are included in the experiment, the noise factor condition for each individual experiment is also determined.

The experiment so designed requires a fixed number of individual experiments, called trial conditions, to be carried out. Depending on the need for variability and noise effects, each trial is tested using multiple samples. When possible, all experiments are carried out in random order. The most common practice is to randomize the order of running the trial conditions and complete all sample tests for the trial in sequence (called *repetition*).

The results of the Taguchi experiments are analyzed in a standard set of phases. First, the factorial effects (main effects) are evaluated, and the influence of the factors is determined in qualitative terms. The optimum condition and the performance at the optimum condition are also determined from the factorial effects. In the next phase, analysis of variance (ANOVA) is performed on the result. ANOVA study identifies the relative influence of the factors to the variation of results in discrete terms. When the experiments include multiple runs and the results are measured in quantitative terms, Taguchi recommends signal-to-noise (S/N) ratio analysis. In S/N analysis, the multiple results of a trial condition are first transformed into S/N ratios and then analyzed.

In the concluding phase of the experimental study, the optimum design identified in the analysis should be tested to confirm that performance observed indeed is the best and that it closely matches the performance predicted (estimated) by analysis.

UP-FRONT THINKING

The value of brainstorming in product development or for solving complex problems is well known, yet it was rarely used for engineering problems. Brainstorming prior to an experiment is a necessary requirement in the Taguchi approach; however, Taguchi does not give any guidelines for conducting brainstorming for an experiment. The content and outcome of a brainstorming session is largely dependent on the nature of a project and, as such, is a technique learned primarily by experience. Most application specialists consider brainstorming to be the most important element in deriving benefits from the Taguchi method.

Taguchi brings a new breadth to planning experimental studies. Experimenters think through the whole process before starting the tests. This helps to decide which factors are likely to be most important, how many experiments are needed, and how the results would be measured and analyzed—before actually conducting any experiment. Figure 4-1 shows the typical steps followed by experimenters—some initial thinking, followed by some testing, which, in turn, is followed by some more thinking, and so on. In the Taguchi approach, the complete plan of how to test, what to test, and when to analyze the results will all be decided beforehand. Ideally, an experiment planning (brainstorming) session will rely on the collective experience of the group to determine the factors to be selected for testing in an appropriate design. Practice of the Taguchi method fosters a team approach to design optimization because participation of people from engineering, manufacturing, testing, and other activities may be necessary for complete variable identification.

EXPERIMENTAL EFFICIENCY

In most cases, the Taguchi experiment design using an orthogonal array requires the least number of test runs. A full factorial experiment with 15 factors at two levels each is performed with a test matrix with 32,768 (2^{15}) test runs. A fractional factorial experiment with an orthogonal array suitable for 15 two-level factors consists of only 16 test runs.

The experimental efficiency Taguchi offers can be described using the following analogy. Assume that you are asked to catch a big fish from a lake with a circular net. You are also told that the fish usually stays around its hideout, but you have no knowledge of where this place is. How do you go about catching this fish? Thinking analytically, you may first calculate the area of the net and the lake and then lay out an elaborate scheme to cover the entire lake. You may find, after all this planning, that you need the whole day to locate the spot where the fish is. Wouldn't it be nice to have a fish finder that could tell you the approximate locations of where to throw your net? The Taguchi approach in experimental studies, to a great extent, works like a fish finder. It tells you which areas to try first, and then from the results of the trials you determine, with a high degree of certainty, the most probable location of the fish.

EFFECTIVE USE OF STATISTICAL PROCESS CONTROL

After design and development comes production. When we complete the Taguchi experimental studies, it is time for statistical process control (SPC). But where do we apply controls? Should we control all factors across the board? If we knew which factors were most significant, it would be wise to pay more attention to them. The information about the relative influence of individual factors to the variability of results is obtained by analysis of variance of the experimental results. This knowledge about the factor influence is used to objectively determine which factors to control and the amount of manufacturing process adjustment necessary.

LONG-TERM BENEFITS

Most of the benefits of quality improvement effort in the design stage come after the product is put in use. The reduced variation, a characteristic that is designed in through the optimum combination of the factors, will yield consistent performance of the product. This means that more of the products will perform as designed. There will be happier customers and, therefore, less warranty costs and increased sales.

QUANTIFYING COST BENEFITS—TAGUCHI LOSS FUNCTION

As indicated earlier, the major attraction of the Taguchi approach is the discipline it introduces in the engineering practices, rather than in direct benefits in time and cost savings. The value of the discipline is extremely hard to quantify. The direct cost savings in terms of a better product or process, on the other hand, can be experienced in due course of time after the product is sold. Can this potential cost savings be estimated before production begins? Dr. Taguchi suggests a way of quantifying such cost savings. He uses his loss function concept to estimate the potential savings based on the improvement achievable if the product were designed to the optimum condition prescribed by his approach.

Suppose XYZ Co., a manufacturer of a 9.00-volt transistor battery, applied the Taguchi method to improve the quality of its product. Before the experiment, a measured sample of 10 batteries had the following voltages:

BEFORE EXPERIMENT				
		Voltages		
8.10	8.25	8.90	8.68	8.35
9.25	9.05	8.85	8.45	8.90

With the target value of 9.00 volts, the above measured values produce characteristics as shown below (also see Tables 4-1 and 4-2).

$$
\begin{aligned}
\text{Average value} &= 8.67 \\
\text{Standard deviation} &= 0.37 \\
\text{Mean square deviation (MSD)} &= [(8.1 - 9.0)^2 + (9.25 - 9.0)^2 \\
&\quad + \ldots + (8.90 - 9.0)^2]/10 \\
&= 0.23 \\
\text{S/N ratio} &= -10 \log_{10} (\text{MSD}) \\
&= 6.36
\end{aligned}
$$

After the experiment, a batch of 10 batteries showed the following characteristics:

AFTER EXPERIMENT				
		Voltages		
9.10	8.93	8.69	8.92	9.08
8.08	9.02	8.91	9.15	9.25
Average value		= 8.99		
Standard deviation		= 0.1598		
Mean square deviation		= 0.023		
S/N ratio		= 16.37		

The signal-to-noise (S/N) ratio expresses the scatter around a target value. The larger the ratio, the smaller the scatter. Taguchi's loss function can be expressed in terms of MSD and, thus, S/N ratios. Knowing the S/N ratios of the samples before and after the experiment, Taguchi's loss function may be used to estimate the potential cost savings from the improved product.

Table 4-1. Standard statistical data before experiment

Observation No.	1	=	8.100
Observation No.	2	=	8.900
Observation No.	3	=	8.450
Observation No.	4	=	9.250
Observation No.	5	=	8.860
Observation No.	6	=	8.350
Observation No.	7	=	8.250
Observation No.	8	=	8.680
Observation No.	9	=	8.900
Observation No.	10	=	9.050

Target/nominal value of result (Y_0)	= 9.00
Number of test results (NR)	= 10
AVERAGE AND STANDARD DEVIATION:	
Total of all test results	= 86.79001
Average of test results	= 8.679001
Standard deviation (SD)	= 0.376252
Variance	= 0.141565
LOSS FUNCTION PARAMETERS:	
Mean square deviation (MSD)	= 0.230449
Signal-to-noise (S/N) ratio	= 6.374235
Variance (modified form)	= 0.127409
Square of mean value	= 0.103041
VARIANCE DATA (ANOVA):	
Target value of data/test result	= 9.00
Mean of data/deviation from target	= –0.321
Total variance (ST)	
(ST = variance * NR)	= 1.274089
Correction factor (CF)	
(CF = (average of data)^2 * number of data)	= 1.030404
Sums of squares/N	= 2.304499

DEFINITIONS:

Standard deviation (SD) = $\sqrt{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2 \Big/ (n-1)}$

Variance = $(SD)^2$

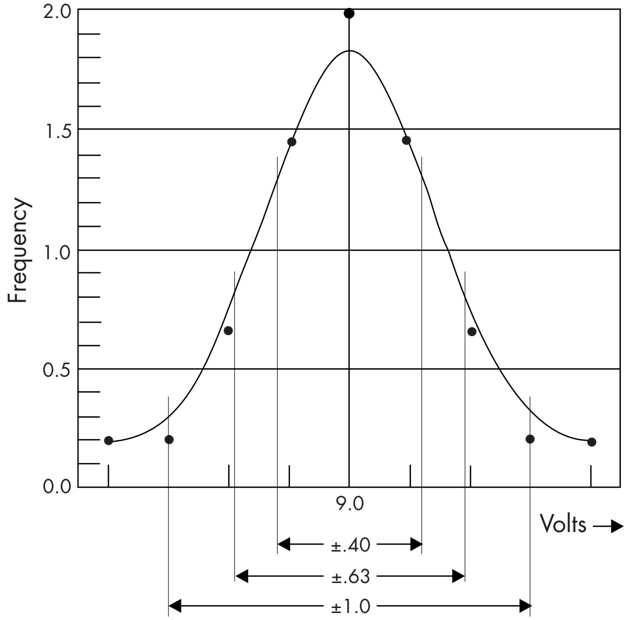
Mean square deviation (MSD) = $\sum_{i=1}^{n}\left(Y_i - Y_0\right)^2 \Big/ n$

Signal/noise (S/N) ratio = $-10 \log_{10}$ (MSD)

Table 4-2. Standard statistical data after experiment

Observation No. 1	=	9.100
Observation No. 2	=	9.080
Observation No. 3	=	8.910
Observation No. 4	=	8.940
Observation No. 5	=	8.880
Observation No. 6	=	9.150
Observation No. 7	=	8.690
Observation No. 8	=	9.020
Observation No. 9	=	9.250
Observation No. 10	=	8.920
Target/nominal value of result (Y_0)	=	9.00
Number of test results (NR)	=	10
AVERAGE AND STANDARD DEVIATION:		
Total of all test results	=	89.93999
Average of test results	=	8.993999
Standard deviation (SD)	=	0.159875
Variance	=	0.025560
LOSS FUNCTION PARAMETERS:		
Mean square deviation (MSD)	=	0.023040
Signal-to-noise (S/N) ratio	=	16.37517
Variance (modified form)	=	0.023040
Square of mean value	=	3.6006E-05
VARIANCE DATA (ANOVA):		
Target value of data/test result	=	9.00
Mean of data/deviation from target	=	–6.00004E-03
Total variance (ST) (ST = variance * NR)	=	0.230040
Correction factor (CF) (CF = (average of data)^2 * number of data)	=	3.6006E-04
Sums of squares/*N*	=	0.230040

Before estimates of savings can be made, some other pertinent information needs to be gathered. Assuming the usual statistical distribution of results, the two samples will exhibit the curve shown in Figure 4-2. The producer, XYZ Co., makes 100,000 units of the batteries per month, which sell for $1.25 each. For most customer applications, the battery voltage should be within ±1.00



COMPUTATION OF MANUFACTURER AND SUPPLIER TOLERANCE

Nominal value of the quality characteristic	Y_0 = 9 volts
Tolerance of *Y* (range of deviation)	= ±1 volt
Cost to repair a nonfunctioning unit by customer	= \$1.25
Cost to repair a nonfunctioning unit by manufacturer	= \$0.5
Cost to repair a nonfunctioning unit by supplier	= \$0.25

REQUIRED TOLERANCES

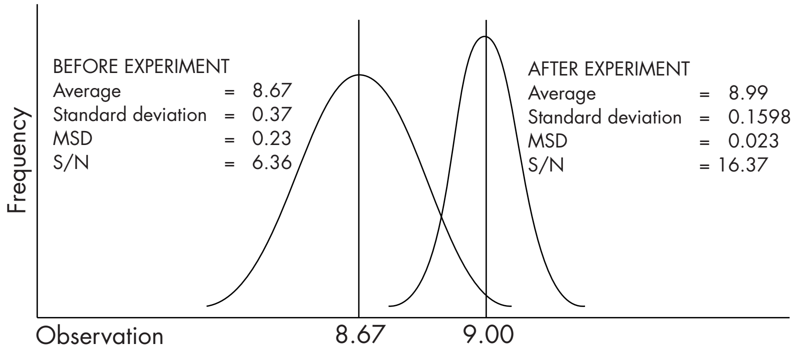
Manufacturer tolerance	= 9 ± 0.63
Supplier tolerance	= 9 ± 0.45

NOTE: If these tolerances are held, there will be no nonfunctional part in the customer's hands. For the same cost, the manufacturer will maintain satisfied customers and quality products in the field.

Figure 4-2. Manufacturer and supplier tolerance

volt, that is, between 8.00 and 10.00 volts. If the voltage is beyond this range, customers request a refund ($1.25).

Taguchi's approach to the computation of cost savings is based on determining the refund cost associated with the variation of the batteries, as measured by the mean square deviation (MSD)



CALCULATION OF LOSS

PROBLEM DEFINITION

Target value of quality characteristic (*m*)	=	9.00
Tolerance of quality characteristic	=	1.00
Cost of rejection at production (per unit)	=	$1.25
Units produced per month (total)	=	100,000
S/N ratio of current design/part	=	6.37
S/N ratio of new design/part	=	16.37

COMPUTATION OF LOSS USING TAGUCHI LOSS FUNCTION

Loss function: $L(y) = 1.25 \times (\text{MSD})$ Also $L(y) = K \times (y \quad m)^2$

BEFORE EXPERIMENT:

Loss/unit due to deviation from target in current design = $0.288

AFTER EXPERIMENT:

Loss/unit due to deviation from target in new design = $0.028

MONTHLY SAVINGS:

If production is maintained at the improved condition, then based on 100,000 units/month = $25,950.90

Figure 4-3. Calculation of cost savings

from the target voltage. Obviously, the greater the variation the more likely that some batteries will exceed the limits of customer acceptance. With the above information, the loss is computed as $.288 per battery for the sample before the experiment and $.028 per unit for the sample after the experiment. Because 100,000 units are manufactured per month, the total savings per month is estimated to be $25,950.90 (Figure 4-3).

SPECIFYING TOLERANCE LEVELS

Another application of Taguchi's loss function formulation is in determining the levels of tolerances for various inspection points of the production process. Suppose that the manufacturer, XYZ Co., is well aware of the losses in the current production samples (before experiment). The company wishes to reduce the warranty costs and expects to keep its customers satisfied. But the company does not want to disturb the current design and production line. It is, however, willing to explore ways to screen out the bad products. The loss function offers some help here.

Let's say XYZ Co., as the manufacturer of batteries, has a producer who supplies the chemicals needed. Upon investigation, XYZ Co. determines that the chemical supplied by the ABC Co. is substandard and is the cause of voltage variation. The manufacturer has two options available. It can inspect the production with the hope to screen out all of the bad products. Or it can ask the supplier to prescreen the material so inspection of the product becomes unnecessary. In either case, the manufacturer needs to establish the limits to which the batteries have to conform. The customer tolerance is established, ± 1.00 volt. Company XYZ must establish a manufacturer tolerance that it will use in the plant, and a tolerance will also be set for the supplier ABC to use for inspection. Taguchi determines these tolerances based on the cost of rejection at the two places. Because the cost of rejecting a finished part is probably greater than the cost of rejection of an ingredient, the tolerances for the manufacturer and the supplier will differ from that of the customers.

In addition to what is already known about the product (battery), two more pieces of information are needed for this calculation:

— cost of rejection/replacement at the manufacturer, and
— cost of rejection/replacement at the supplier.

Suppose that a part costing 20 cents from a supplier can produce a loss of 50 cents to the manufacturer if the part fails. The loss equations will produce tolerances of ± 0.63 volts and ± 0.4 volts for the manufacturer and the supplier, respectively, as shown in Figure 4-2. Either the supplier or the manufacturer can screen the products. To assure defect-free products, the supplier may screen them before they are shipped to the manufacturer, who in turn passes them to the customers. When a supplier doesn't screen, the manufacturer must. (The calculations shown in Figs. 4-2 and 4-3 are obtained by using the computer software in [7].)

EXERCISES

4-1. The Taguchi method is considered a technique that helps build quality into a product or process. Explain what aspect of quality it influences and how.

4-2. Compare the roles of the Taguchi method with that of statistical process control (SPC) in a manufacturing process. Explain how the Taguchi method can influence decisions in the SPC activities.

5 Working Mechanics of the Taguchi Design of Experiments

FORMULAS FOR EXPERIMENT LAYOUT

It should be quite clear by now that the Taguchi method is intended for improving the quality of products and processes where the performance depends on many factors. In laying out a test and development strategy, simple logic will usually be sufficient to establish all possible combinations of factors along with allowable ranges of each of the factors involved. Unfortunately, for engineering projects involving many factors, the number of possible combinations is prohibitively large. In addition, higher-order interactions among the influencing factors may be needed for specific projects. A customary method of reducing the number of test combinations is to use what are known as partial (or fractional) factorial experiments. To secure more economical test plans, Dr. Taguchi constructed a special set of general designs for factorial experiments that cover many applications. The special set of designs consists of tables of numbers called orthogonal arrays (OAs). The use of these arrays helps determine the least number of experiments needed for a given set of factors. The details of using standard (not modified) OAs in designing experiments for a given set of factors is the subject of this chapter.

The OAs provide a recipe for fractional factorial experiments, which satisfy a number of situations. When a fixed number of levels for all factors is involved and the interactions are unimportant, standard OAs will satisfy most experimental design needs. A modification of the OAs becomes necessary when factors with mixed levels and interactions are present. Simple designs with a smaller number of factors, at fixed levels, will be discussed first.

BASIC METHODOLOGY

The technique of laying out the conditions (designs) of experiments involving multiple factors was first proposed by Sir Ronald A. Fisher of England in the 1920s. The method is popularly known as the factorial design of experiments. A full factorial design will identify all possible combinations for a given set of factors. Because most industrial experiments usually involve a significant number of factors, a full factorial design results in a large number of experiments. For example, in an experiment involving seven factors, each at two levels, the total number of combinations will be 128 (2^7). To reduce the number of experiments to a practical level, only a small set from all of the possibilities is selected. The method of selecting a limited number of experiments that produces the most information is known as a fractional factorial experiment. Although this shortcut method is well known, there are no general guidelines for its application or the analysis of the results obtained by performing the experiments. Dr. Taguchi's approach complements these two important areas. First, he clearly defined a set of OAs, each of which can be used for many experimental situations. Second, he devised a standard method for analysis of the results. The combination of standard experimental design techniques and analysis methods in the Taguchi approach produces a higher degree of consistency and reproducibility of the predicted performance.

Before discussing how the Taguchi approach reduces the number of experiments, it is helpful to understand how all possible combinations result from a set of factors.

Suppose we are concerned about one factor, A (say, temperature). If we were to study the effect of A on a product at two levels, say, 400°F and 500°F, then two tests become necessary:

Level 1 = A_1 (400°F) and Level 2 = A_2 (500°F)

Consider now two factors, A and B, each at two levels (A_1, A_2 and B_1, B_2). This produces four combinations because at A_1, B can assume values B_1 and B_2, and at A_2, B can again assume values B_1 and B_2.

Symbolically, these combinations are expressed as follows:

$A_1(B_1, B_2)$, $A_2(B_1, B_2)$, or as A_1B_1, A_1B_2, A_2B_1, A_2B_2

With three factors, each at two levels, there are 2^3 (8) possible experiments, as described in the previous section. If A, B, and C represent these factors, the eight experiments can be expressed as follows:

$A_1B_1C_1$, $A_1B_1C_2$, $A_1B_2C_1$, $A_1B_2C_2$, $A_2B_1C_1$, $A_2B_1C_2$, $A_2B_2C_1$, and $A_2B_2C_2$

Using the above general rule, the total number of experiments possible for different numbers of factors at two or three levels and the corresponding suggested Taguchi number of experiments are shown in Table 5-1.

A factorial experiment of seven factors (A, B, C, D, E, F, G), at two levels of value each (1 and 2), with 128 possible combinations, is represented by Table 5-2(a). Each of the 128 cells corresponds to a unique combination of the factors. As shown in Table 5-2(b), cells T1 through T8 indicate the eight trial numbers defined by Taguchi's fractional factorial OA for this experiment.

Taguchi established OAs that can each be used to lay out tests suitable for a large number of experimental situations. The symbolic designation for these arrays carries the key information on the size of the experiment. The array of Table 5-2(b) is designated as L-8 or L_8. The number 8 indicates that eight trials are needed. The next lower size of the OA is L_4. An L_4 experiment requires four trial runs. This array handles up to three factors at two levels each. To fit a situation with factors between four and seven, all at

Table 5-1. Comparison of full factorial design and Taguchi design

		TOTAL NUMBER OF EXPERIMENTS	
FACTORS	LEVELS	FULL FACTORIAL DESIGN	TAGUCHI DESIGN
2	2	4 (2^2)	4
3	2	8 (2^3)	4
4	2	16 (2^4)	8
7	2	128 (2^7)	8
15	2	32,768 (2^{15})	16
4	3	81 (3^4)	9

Table 5-2. Experiment layouts using an L_8 array

(a) Experiment structure

FULL FACTORIAL EXPERIMENTS				A_1				A_2			
				B_1		B_2		B_1		B_2	
				C_1	C_2	C_1	C_2	C_1	C_2	C_1	C_2
D_1	E_1	F_1	G_1	T1							
			G_2								
		F_2	G_1								
			G_2				T3				
	E_2	F_1	G_1								
			G_2						T5		
		F_2	G_1							T7	
			G_2								
D_2	E_1	F_1	G_1								
			G_2							T8	
		F_2	G_1						T6		
			G_2								
	E_2	F_1	G_1				T4				
			G_2								
		F_2	G_1								
			G_2	T2							

(b) Trial runs and conditions

COLUMN	1	2	3	4	5	6	7
FACTOR	*A*	*B*	*C*	*D*	*E*	*F*	*G*
TRIAL							
T1	1	1	1	1	1	1	1
T2	1	1	1	2	2	2	2
T3	1	2	2	1	1	2	2
T4	1	2	2	2	2	1	1
T5	2	1	2	1	2	1	2
T6	2	1	2	2	1	2	1
T7	2	2	1	1	2	2	1
T8	2	2	1	2	1	1	2

two levels, an L_8 will be used. For situations demanding a larger number of factors—higher levels as well as mixed levels, a number of other OAs are available [10].

Experiment designs by OAs are attractive because of experimental efficiency, but there are some potential trade-offs. Generally speaking, OA experiments work well when there is minimal interaction among factors; that is, the factor influences on the measured quality objectives are independent of each other and are linear. In other words, when the outcome is directly proportional to the linear combination of individual factor main effects, OA design identifies the optimum condition and estimates performance at this condition accurately. If, however, the factors interact with each other and influence the outcome, there is still a good chance that the optimum condition will be identified accurately, but the estimate of performance at the optimum can be significantly off. The degree of inaccuracy in performance estimates will depend on the degree of complexity of interactions among all the factors.

DESIGNING THE EXPERIMENT

The word "design" in the expression design of experiments is used in a general sense to convey "a planned project or a scheme in which the means to an end are laid down." To design the experiment is to develop a scheme or layout of the different conditions to be studied. In engineering, the word takes on a special meaning when used as "a design," "product design," or "process design." In these expressions, design refers to some form of an engineering communication, such as a set of specifications, drawings, or physical models that describe a concept. Consider a statement like, "The Taguchi design of experiments can be used to optimize many designs." The final "design" in this sentence obviously refers to some engineering design process.

An experiment design must satisfy two objectives. First, the number of trials must be determined. Second, the conditions for each trial must be specified. Taguchi's arrays are versatile recipes that apply to several experimental conditions. For example, the design for experiments involving 4, 5, 6, or 7 two-level factors may all be accomplished by using the same orthogonal array (L_8).

The OAs contain information on both the number as well as the configurations of the experiments.

Before designing an experiment, knowledge of the product/ process under investigation is of prime importance for identifying the factors likely to influence the outcome. To compile a comprehensive list of the factors, the input to the experiment is generally obtained from all of the people involved in the project. Dr. Taguchi found brainstorming to be a necessary step for determining the full range of factors to be investigated.

Consider *Example 5-1*.

Example 5-1

An experimenter has identified three controllable factors for a plastic molding process. Each factor can be applied at two levels (Table 5-3). The experimenter wants to determine the optimum combination of the levels of these factors as well as the contribution of each to product quality.

Experiment Design

There are three factors, each at two levels, thus an L_4 will be suitable, per Table 5-1. An L_4 OA with spaces for the factors and their levels is shown in Table 5-4. This configuration is a convenient way to lay out a design. Because an L_4 has three columns, the three factors can be assigned to these columns in any order. Having assigned the factors, their levels can also be indicated in the corresponding column.

There are four independent experimental conditions in an L_4. These conditions are described by the numbers in the rows. For an experienced user of the technique, an array with factors assigned as shown in Table 5-2 contains all of the necessary information;

Table 5-3. Molding process factors and levels—Example 5-1

FACTOR	LEVEL 1	LEVEL 2
A. Injection pressure	A_1 = 250 psi	A_2 = 350 psi
B. Mold temperature	B_1 = 150°F	B_2 = 200°F
C. Set time	C_1 = 6 sec.	C_2 = 9 sec.

for others, a descriptive arrangement of the factors constituting different conditions of the experiment may be helpful. In this case, the four conditions can be spelled out as follows:

Experiment 1		
Injection pressure	at	250 psi, that is, A_1
Mold temperature	at	150°F, that is, B_1
Set time	at	6 sec., that is, C_1
Experiment 2		
Injection pressure	at	250 psi, that is, A_1
Mold temperature	at	200°F, that is, B_2
Set time	at	9 sec., that is, C_2
Experiment 3		
Injection pressure	at	350 psi, that is, A_2
Mold temperature	at	150°F, that is, B_1
Set time	at	9 sec., that is, C_2
Experiment 4		
Injection pressure	at	350 psi, that is, A_2
Mold temperature	at	200°F, that is, B_2
Set time	at	6 sec., that is, C_1

Table 5-4. Experiment layout using an L_4 array—Example 5-1

VARIABLE DESCRIPTION	LEVEL 1	LEVEL 2
Injection pressure	250 psi	350 psi
Mold temperature	150°F	200°F
Set time	6 sec.	9 sec.

EXPERIMENT	COLUMN 1	COLUMN 2	COLUMN 3	REPETITION 1	2	3	• • •
1	1	1	1	30			
2	1	2	2	25			
3	2	1	2	34			
4	2	2	1	27			

Order of Running the Experiments

Whenever possible, the trial conditions (the individual combinations in a designed experiment) should be run in a random order to avoid the influence of experiment setup. If only one run for each of the above conditions is planned, they could be run as experiment 2, 4, 3, and 1, or in any other random order. If, on the other hand, multiple repetitions are planned, say three runs for each of the four conditions, then there are two ways to proceed.

Replication

In this approach, all of the trial conditions will be run in a random order. One way to decide the order is to randomly pull one trial number at a time from a set of trial numbers, including repetitions. Often a new setup will be required for each run. This increases the cost of the experiment.

Repetition

Each trial is repeated as planned before proceeding to the next trial run. The trial run sequence is selected in a random order. For example, given the trial sequence 2, 4, 3, and 1, three successive runs of trial 2 are made, followed by three runs of trial 4, and so on. This procedure reduces setup costs for the experiment. However, a setup error is unlikely to be detected. Furthermore, the effect of external factors such as humidity, tool wear, and so on, may not be captured during the successive runs if the runs are short in duration.

Analysis of Results

Although, a detailed analysis of the results will be discussed in Chapter 6, a brief description and objectives of such an analysis are introduced here.

Following the specifications as prescribed above, the experimenter conducted the four trials. The molded products were then evaluated, and the results, in terms of a quality characteristic, Y, were measured as shown below:

$$Y_1 = 30,\ Y_2 = 25,\ Y_3 = 34,\ Y_4 = 27$$

These results are recorded in the right-most column of the OA (Table 5-5). Because there was only one test sample in each trial condition, the results are recorded in one column. For each repetition of the experiment, there will be another column of results.

To speed up analysis, the Taguchi approach provides some key procedures. When these steps are strictly followed by different individuals performing the analysis, they are likely to arrive at the same conclusions. The objective of the analysis of the Taguchi experimental results is primarily to seek answers to the following three key questions:

1. What is the optimum condition?
2. Which factors influence the variability of results and by how much?
3. What will be the expected result at the optimum condition and how much does each factor contribute to the improvement?

The predicted result should always be verified by running confirmation experiments.

Computation of Average Performance

To compute the average performance of factor A at level 1, that is, for A_1, add results (from Table 5-3) for trials including factor A_1 and then divide by the number of such trials.

For A_1, we look in the column where factor A is assigned and find that level 1 occurs in trials 1 and 2. The average effect of A_1 is therefore calculated by adding the results, Y, of these two trials as follows:

Table 5-5. An L_4 array with test data of molding process experiment

FACTOR TRIAL	*A*	*B*	C	RESULT (*Y*)
1	1	1	1	30
2	1	2	2	25
3	2	1	2	34
4	2	2	1	27

$$\bar{A}_1 = (Y_1 + Y_2)/2 = (30 + 25)/2 = 27.5$$

The average effects of other factors are computed similarly.

$$\bar{A}_2 = (Y_3 + Y_4)/2 = (34 + 27)/2 = 30.5$$
$$\bar{B}_1 = (Y_1 + Y_3)/2 = (30 + 34)/2 = 32.0$$
$$\bar{B}_2 = (Y_2 + Y_4)/2 = (25 + 27)/2 = 26.0$$
$$\bar{C}_1 = (Y_1 + Y_4)/2 = (30 + 27)/2 = 28.5$$
$$\bar{C}_2 = (Y_2 + Y_3)/2 = (25 + 34)/2 = 29.5$$

The average effects can also be plotted for a visual inspection, as shown in Figure 5-1. Frequently, the term "factorial effect," or "main effect" or "column effect," is loosely substituted for "average effect." Strictly speaking, the factorial effect is the difference between the two average effects of the factor at the two levels. For instance, the factorial effect of factor C is the difference between the average effect of C_1 and C_2.

Quality Characteristics

In a previous chapter, the quality characteristics were described as:

- bigger is better
- smaller is better
- nominal is best

For the molding process example, higher strength of the molded plastic part is desired and thus "bigger is better." From Figure 5-1, the $A_2\ B_1\ C_2$ will likely produce the best result and therefore represents the optimum condition except for the possible effect of interactions between the factors.

In terms of the actual design factors, the probable optimum condition becomes:

A_2	that is, injection pressure	at	350 psi
B_1	that is, mold temperature	at	150°F
C_2	that is, set time	at	9 sec.

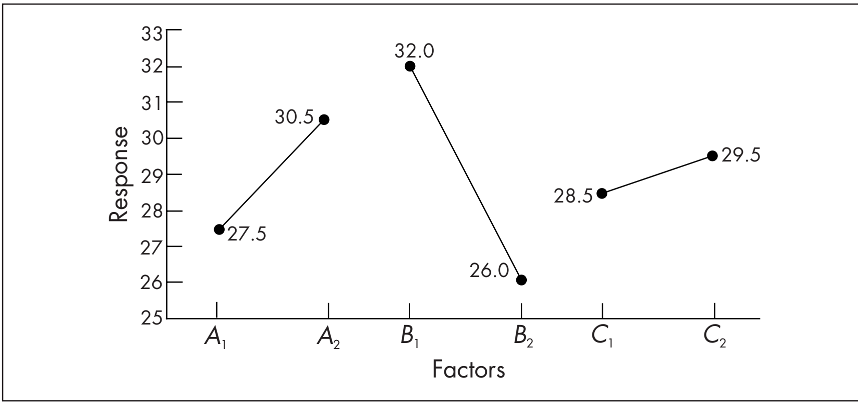


Figure 5-1. Main effects

Relative Influence of Factors

The relative influences of factors to the variation of results are determined by comparing their variances. The technique popularly known as the analysis of variance (ANOVA) is used for this purpose. ANOVA will be covered in detail in Chapter 6. Here the procedure is briefly introduced to complete the analysis.

ANOVA Terms and Notations

The analysis of variance computes quantities known as degrees of freedom, sums of squares, mean square, and so on, and organizes them in a standard tabular format. These quantities and their interrelationships are defined as shown below using the following notation:

V	= mean square (variance)	P	= percent influence
S	= sum of squares	T	= total (of results)
S'	= pure sum of squares	N	= number of experiments
f	= degrees of freedom	C.F.	= correction factor
e	= error (experimental)	n	= total degrees of freedom
F	= variance ratio		

Variance

The variance of each factor is determined by the sum of the square of each trial sum result involving the factor, divided by the degrees of freedom of the factor. Thus:

$V_A = S_A / f_A$ (for factor A)

$V_B = S_B / f_B$ (for factor B)

$V_C = S_C / f_C$ (for factor C)

$V_e = S_e / f_e$ (for error terms)

Variance Ratio

The F-ratio is the variance of the factor divided by the error variance.

$F_A = V_A / V_e$

$F_B = V_B / V_e$

$F_C = V_C / V_e$

$F_e = V_e / V_e = 1$

Pure Sum of Squares

The pure sum of squares is the sum minus the degrees of freedom times the error variance.

$S'_A = S_A - f_A \times V_e$

$S'_B = S_B - f_B \times V_e$

$S'_C = S_C - f_C \times V_e$

$S'_e = S_e + (f_A + f_B + f_C) \times V_e$

Percent Influence

The percent influence of each factor is the ratio of the factor sum to the total, expressed in percent.

$P_A = S_A \times 100 / S_T$

$P_B = S_B \times 100 / S_T$

$P_C = S_C \times 100 / S_T$

$P_e = S_e \times 100 / S_T$

Examples of Variation Computation

Total variation: S_T = Sum of squares of all trial run results – *C.F.*

where: $C.F. = T^2/N$ and $T = (Y_1 + Y_2 + Y_3 + Y_4)$

$$\text{or } S_T = (Y_1^2 + \cdots + Y_4^2) - (Y_1 + Y_2 + Y_3 + Y_4)^2/4$$
$$= 30^2 + 25^2 + 34^2 + 27^2 - (30 + 25 + 34 + 27)^2/4$$
$$= 3410 - 3364$$
$$= 46$$

For the molding process experiment, the totals of the factors are:

$$A_1 = 30 + 25 = 55 \qquad A_2 = 34 + 27 = 61$$
$$B_1 = 30 + 34 = 64 \qquad B_2 = 25 + 27 = 52$$
$$C_1 = 30 + 27 = 57 \qquad C_2 = 25 + 34 = 59$$

therefore, the total variance of each factor is:

$$S_A = A_1^2/N_{A1} + A_2^2/N_{A2} - C.F.$$
$$= 55^2/2 + 61^2/2 - 3364$$
$$= 1512.5 + 1860.5 - 3364 = 9.0$$
$$S_B = B_1^2/N_{B1} + B_2^2/N_{B2} - C.F. = 36.0$$

and

$$S_C = C_1^2/N_{C1} + C_2^2/N_{C2} - C.F. = 1\ 0$$

The error variance

$$S_e = S_T - (S_A + S_B + S_C)$$
$$= 46 - 9 - 36 - 1 = 0 \text{ (in this case)}$$

Degrees of Freedom (DOF)

The number of the degrees of freedom for a factor or a column equals one less than the number of levels. Thus, for a two-level factor assigned to a two-level column, the DOF is 1. An L_4 OA with three two-level columns will have a total of 3 DOF, or one for each column. The total degrees of freedom of the result T, however, is computed as follows:

f_T = total number of results – 1
 = (total number of trials × number of repetition) – 1
 = 4 × 1 – 1 = 3

For factors

f_A = number of levels of A – 1 = 1
f_B = number of levels of B – 1 = 1
f_C = number of levels of C – 1 = 1

and the DOF for error variance is

$$f_e = f_T - f_A - f_B - f_C$$
$$= 3 - 1 - 1 - 1$$
$$= 0$$

Variance

$$V_A = S_A/f_A = 9/1 = 9$$
$$V_B = S_B/f_B = 36/1 = 36$$
$$V_C = S_C/f_C = 1/1 = 1$$
$$V_e = S_e/f_e = 0/0 \text{ indeterminate}$$

Note that if the experiment included repetitions, say 2, then:

$$f_T = 4 \times 2 - 1 = 7$$
$$f_e = 7 - 1 - 1 - 1 = 4$$

where S_e need not equal zero, depending on test results, and V_e need not be zero.

Variance Ratio

$F_A = V_A/V_e$ is indeterminate because $V_e = 0$. Similarly, F_B and F_C are indeterminate (Table 5-6). However, V_e can be combined (pooled) with another small variance, V_C, to calculate a new error V_e that can then be used to produce meaningful results. The process of disregarding an individual factor's contribution and then subsequently adjusting the contributions of the other factors is known as *pooling*. Generally, only factors that are believed to be insignificant are pooled. Whether a factor is significant or not is found by the test of significance. The detailed procedure for the test of significance (for pooling) and the criteria used to determine its use will be discussed in Chapter 6 along with ANOVA.

Consider the pooled effects of factor C. Then the new error variance is computed as:

$$V_e = (S_C + S_e)/(f_C + f_e)$$
$$= (1.0 + 0)/(1.0 + 0) = 1$$

With this new V_e, all sums of squares, S, can be modified as

$$S'_A = S_A - (V_e \times f_A)$$

and so on.

The result then can be shown in the ANOVA table with the effect of factor C pooled. The pooled effects are shown as the error term in the ANOVA table (last row of Table 5-7).

The last column of the ANOVA table shows the percent contribution of the individual factor. In the example, factor B contributes the most, 76.08%. The contribution of A is 17.39% and that of C is not significant.

Note that the difference in the percentage influences of factors before (Table 5-6) and after pooling (Table 5-7) is not large. To increase the statistical significance of important factors, those factors with small variances should be pooled.

Projection of Optimum Performance

Recall that for a "bigger is better" quality characteristic, the study of the main effect shows that the optimum condition is $A_2 B_1 C_2$. It happens to be the third trial run. This is just a coincidence. Most of the time, the optimum condition will not be one of the trial runs because a Taguchi experiment represents only a small set of the full factorial experiment. The probability is 50% that the optimum condition is one of tests carried out. This is because in an L_4 experiment four out of eight full factorial conditions are tested. Of course, regardless of the size of the experiment, the

Table 5-6. ANOVA table for molding process experiment

NOTATION FACTOR	f	S	V	F	s	P (%)
A	1	9	9	—	—	19.62
B	1	36	36			78.28
C	1	1	1			2.10
Error	0	0	0			
Total	3	46				100.00%

Table 5-7. Pooled ANOVA table for molding process experiment

NOTATION SOURCE	f	S	V	F	s	P (%)
A	1	9	9	9	8	17.39
B	1	36	36	36	35	76.08
C	--------	pooled	--------	--------	--------	--------
Error	1	1	1			6.74
Total	3	46				100.00%

optimum is always one of the trials defined by the full factorial experiment. As a general rule, the optimum performance will be estimated using the following expression.

T = grand total of all results
N = total number of results
Y_{opt} = estimated performance at optimum condition

For optimum combination $A_2 B_1 C_2$ (which happens to be experiment 3)

$$Y_{opt} = T/N + \left(\bar{A}_2 - T/N\right) + \left(\bar{B}_1 - T/N\right) + \left(\bar{C}_2 - T/N\right)$$

= average performance + contribution of A_2, B_1, and C_2 above average performance

In this example:

$$T = 116,\ N = 4,\ \bar{A}_2 = 30.5,\ \bar{B}_1 = 32,\ \bar{C}_2 = 29.5$$

therefore,

$$Y_{opt} = 29 + (30.5 - 29) + (32 - 29) + (29.5 - 29)$$
$$= 34.0$$

which is the result obtained in trial 3.

When the optimum is not one of the trial runs already completed, this projection should be verified by running a confirmation test(s). Confirmation testing is a necessary and important step in the Taguchi method as it validates assumptions used in the analysis. Generally speaking, the average result from the confirmation tests should agree with the optimum performance, Y_{opt}, estimated by the analysis. The correlation can also be established in statistical terms, reflecting the level of confidence, influence of number of confirmation tests, and so on. The procedure for calculating the confidence interval of the optimum performance is discussed in Chapter 6.

DESIGNING WITH MORE THAN THREE VARIABLES

In the preceding section, layout of a simple experiment involving only three factors was discussed. In this section, designs with a larger number of factors will be considered. The designs will use the higher-order orthogonal arrays (OAs).

Designs with Two-Level Variables

Example 5-2

Design an experiment to investigate

	Four factors	all at two levels,
	Five factors	all at two levels,
	Six factors	all at two levels,
or	Seven factors	all at two levels.

Let these factors be A, B, C, D, E, F, and G and their levels be A_1, A_2, and so on.

Experiment Design

As seen in the last example, the smallest OA, L_4, can handle up to three factors. What if there are more than three factors?

Table 5-8. L_8 with seven two-level factors—Example 5-2

FACTOR	A	B	C	D	E	F	G	
COLUMN	1	2	3	4	5	6	7	
EXPERIMENT								RESULT
1	1	1	1	1	1	1	1	
2	1	1	1	2	2	2	2	
3	1	2	2	1	1	2	2	
4	1	2	2	2	2	1	1	
5	2	1	2	1	2	1	2	
6	2	1	2	2	1	2	1	
7	2	2	1	1	2	2	1	
8	2	2	1	2	1	1	2	

A list of commonly used OAs is shown in Table A-1. Notice that L_8 can be used for four to seven factors. Therefore, L_8 is suitable for any of the above situations.

An L_8 array has eight trial conditions and seven columns. To design the experiment, factors must be assigned to appropriate columns, and eight trial conditions must be described.

Because all factors have the same number of levels, the factor can be assigned to any one column. Thus, factor A can be assigned to any of the columns 1 through 7. Then B can be assigned to any one of the remaining columns. You can also assign them in naturally ascending order, like A in column 1, B in column 2, and so on. If there are only four factors, ignore the three unused columns.

The experimental conditions are defined by reading across the row of the OA. An L_8 OA, as shown in Table A-2, has eight rows. Thus, it represents eight unique combinations of factors and their levels. An L_8 with the factors assigned to its columns is shown in Table 5-8.

From an L_8 array, trial 3 is defined as:

Trial 3 $\quad A_1 \quad B_2 \quad C_2 \quad D_1 \quad E_1 \quad F_2 \quad G_2$
(the numbers in the OA represent the levels of the factor assigned to the column)

Example 5-3

Number of factors	=	8 through 11
Number of levels for each	=	2

Use array L_{12}

Experiment Design

Use L_{12} in Table A-3 for this example. Assign factors 1 through 11 in the 11 columns available, in any order. Express the 12 experimental conditions by using the 12 rows of the OA. Note that L_{12} is a special array prepared for study of the main effects only (not suitable for study of interaction between factors). In this array, the interaction effects of factors assigned to any two columns are mixed with all other columns, which renders the array unsuitable for interaction studies. (Use L_{16}, L_{32}, and L_{64} shown in Appendix A to design experiments with higher numbers of two-level factors.)

Designs with Three-Level Variables

Example 5-4

Number of factors	=	5 through 13
Number of levels for each	=	3

Use array L_{27} (Table A-8)

Designs with Mixed Levels Using Standard Arrays

Example 5-5

Consider the experiments where there are eight factors to be investigated. To study the nonlinear effect, seven factors were set at three levels. The remaining one factor was examined at only two levels.

Experiment Design

This is an example of mixed levels. The L_{18} array shown in Table A-7(b) is one of the few standard mixed-level arrays and is used in this case. L_{18} has eight columns, with column 1 having two levels and the rest having three levels. Obviously, the factor with two levels will be assigned to column 1. The remaining seven factors can be assigned to columns 2 through 8 in any desired manner.

DESIGNS WITH INTERACTION

The term *interaction*, expressed by inserting an "×" mark between the two interacting factors, is used to describe a condition in which the influence of one factor on the result is dependent on the condition of the other. Two factors, A and B, are said to interact (written as $A \times B$) when the effect of changes in the level of A determine the influence of B, and vice versa.

For example, temperature and humidity appear to have strong interaction with respect to human comfort. An increase in temperature alone may cause slight discomfort, but the discomfort increases as humidity increases. Assume the comfort level is dependent only on two factors, T and H, and is measured in terms of numbers ranging from 0 to 100. If T and H are each allowed to assume levels as T_1, T_2, H_1, and H_2, assume that two sets of experimental data (with the same grand total of all observations) are obtained and represented by Tables 5-9(a) and (b). The data are plotted in Figures 5-2(a) and (b). Figure 5-2(a) shows an interaction between the two factors because the lines cross. Figure 5-2(b) shows no interaction because the lines are parallel. If the lines are not parallel, the factors may interact, albeit weakly.

The graphical method reveals if interaction exists. The input for this interaction plot comes from the experimental results, and the degree of presence of interaction is calculated as the magnitude of the angle between the lines. But how can we know whether the factors will interact before we design the experiment? The Taguchi methods do not specify any general guidelines for predicting interactions. One has to determine interaction by some other means, perhaps from experience or previous experimental studies.

Experimental design using Taguchi OAs is simple and straightforward when there is no need to include interactions. It requires a little more care to design an experiment where interactions are of interest and are included in the study. In Taguchi OAs, the effect of interactions are mixed with the main effect of a factor assigned to some other column. In the L_4 shown in Table 5-10 with factors A and B assigned to columns 1 and 2, interaction effects of $A \times B$ will be contained in column 3. If the interactions of $A \times B$ are

Table 5-9. Layout for experiment with two two-level factors

(a) Case with interaction

	T_1	T_2	TOTAL
H_1	62	80	142
H_2	75	73	148
Total	137	153	290

(b) Case without interaction

	T_1	T_2	TOTAL
H_1	67	75	142
H_2	70	78	148
Total	137	153	290

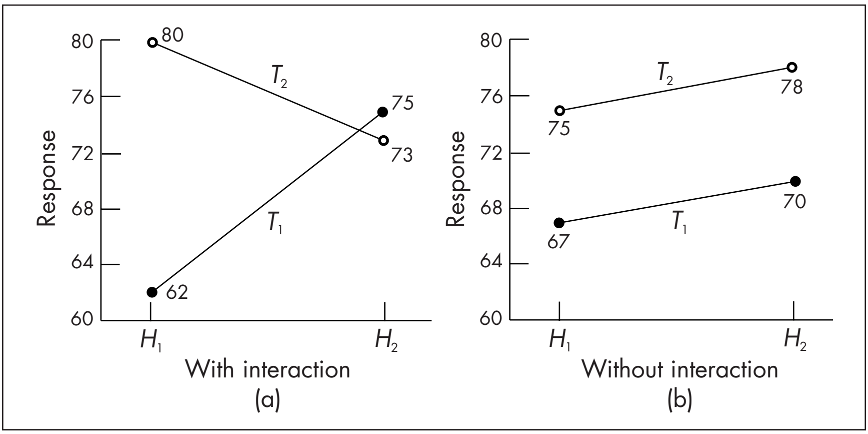


Figure 5-2. Main effects of factors T and H

Table 5-10. L_4 with two two-level factors

COLUMN	1	2	3
FACTOR	A	B	$A \times B$ C
EXPERIMENT			
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

of no interest, a third factor C can be assigned to column 3 (see Table 5-10). The effect of interaction $A \times B$ will then be mixed with the main effect of factor C.

The interacting pair of columns along with the column where the interaction is shown constitute an *interacting group of columns* (IGCs). An IGC consists of columns that are commutative. Columns 1, 2, and 3 form one of the IGCs. In case of an L_8 OA, the interaction of columns 1 and 2 goes to column 3. This also means that the interaction between 2 and 3 is reflected in column 1 and that between 3 and 1 shows up in 2. What about interactions between other columns of an L_8 or any other two-level orthogonal arrays? What about interaction between two columns of three-level orthogonal arrays? These are difficult tasks for practicing engineers to keep track of. Dr. Taguchi spent much of his research determining relationships for interacting columns. His findings regarding which columns interact with which others are presented in a table called the *Triangular Table of Interaction*. There are tables to suit different levels of OAs. A large triangular table as shown in Table A-6, made for two-level columns, will usually satisfy most commonly used two-level orthogonal arrays.

Note that the first seven columns of Table A-6 become the triangular table for L_8, as shown in Table 5-11.

A triangular table contains information about the interaction of the various columns of an OA. The table should be interpreted in the following way. The number in parentheses at the bottom of each column identifies the column. To find in which column the interaction between columns 4 and 6 will appear, move horizontally across 4 and vertically from 6; the intersection is 2 in

Table 5-11. Interaction between two columns in an L_8 array

COLUMN	1	2	3	4	5	6	7
	(1)	3	2	5	4	7	6
		(2)	1	6	7	4	5
			(3)	7	6	5	4
				(4)	1	2	3
					(5)	3	2
						(6)	1
							(7)

the tables. Thus, the interaction effects between columns 4 and 6 will appear (mixed or confounded with factors) at column 2. In a similar manner, other interacting columns can be identified.

The triangular table facilitates laying out experiments with interactions. The table greatly reduces the time and increases the accuracy of assigning proper columns for interaction effects. To further enhance efficiency of the experimental layout, Taguchi created line diagrams based on the triangular tables. These diagrams represent standard experiment designs. He calls such diagrams *linear graphs*. Linear graphs for L_4, L_8, and other two-level orthogonal arrays are shown in Figures A-2 and A-3.

Linear Graphs

Linear graphs are made up of numbers, dots, and lines, as shown in Figure 5-3 for an L_4 array, where a dot and its assigned number identifies a factor, a connecting line between two dots indicates interaction, and the number assigned to the line indicates the column number in which interaction effects will be compounded.

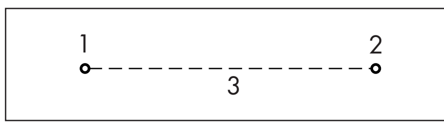


Figure 5-3. Linear graph for L_4 array

In designing experiments with interactions, the triangular tables are essential; the linear graphs are complementary to the tables. For most industrial experiments, interactions between factors are minor and the triangular tables suffice. The following example shows how these two tools are used for experimental design.

Example 5-6

In a baking experiment designed to determine the best recipe for a pound cake, five factors and their respective levels were identified, as presented in Table 5-12.

Among these factors, milk (factor C) was suspected to interact with eggs (A) and butter (B). An experiment was designed to study the interactions $A \times C$ and $B \times C$ in addition to the main effects of factors A, B, C, D, and E.

Table 5-12. Factors for cake baking experiment—Example 5-6

FACTOR	LEVEL 1	LEVEL 2
A—Eggs	2 eggs (A_1)	3 eggs (A_2)
B—Butter	1 stick (B_1)	1.5 sticks (B_2)
C—Milk	2 cups (C_1)	3 cups (C_2)
D—Flour	1 extra scoop (D_1)	2 extra scoops (D_2)
E—Sugar	1 extra scoop (E_1)	2 extra scoops (E_2)

Steps in the Design and Analysis

Degrees of Freedom (DOF)

Each of the five factors (A, B, C, D, E) is to be studied at two levels; therefore, each factor has a DOF of 1 (DOF = number of levels – 1). The DOF for the interaction is computed by multiplying the DOF of each of the interacting factors. Thus, the DOF for $A \times C$ is 1. Likewise, the DOF of $B \times C = 1$. The total DOF for the five factors and two interactions in this case is 7. The appropriate Taguchi array cannot have a DOF less than the total DOF of the experiment.

Selecting the Right Orthogonal Array

The experiment under consideration has 7 DOF and therefore requires an OA with 7 DOF, hence an OA with at least seven columns. Because an L_4 has three columns, its DOF is 3. An L_8 has seven columns and 7 DOF; it possibly can work. An L_{12} has 11 two-level columns, of which only seven are needed. L_{12} certainly will work but will require 12 trial runs in contrast to eight for L_8. The smallest OA that will do the job should be selected to minimize experiment cost and time. How about an L_{16}? That is too large for a 7 DOF experiment. In this case, an L_8 is a good match. Would an L_8 always work for an experiment with 7 DOF? Not necessarily. It will depend mainly on how many interactions are expected to be investigated. It will work for *Example 5-6*.

Column Assignment

In designing experiments with interactions, the columns to reserve to study interactions must be identified first. We have two interactions, $A \times C$ and $B \times C$. The trick is to select positions for $A \times C$ and $B \times C$ such that there are free columns for each of the factors A, B, and C as well. This can be done by using the triangular table for a two-level OA or the corresponding linear graphs. Let us examine the linear graph (a) of Figure A-2. C is common to $A \times C$ and $B \times C$. Assign C to column 2, a vertex with two connecting lines. Notice column 2 is a vertex of the triangle with sides 2-3-1 and 2-6-4. With C at 2, assign A to either column 1 or column 4 and B to any remaining column. If A is assigned to 1 and B to 4, then $A \times C$ becomes column 3 and $B \times C$ becomes column 6.

Five columns have been used by factors A, B, and C and interactions $A \times C$ and $B \times C$. The remaining two factors, D and E, can be assigned to columns 5 and 7 in any order. Let us assign D to column 5 and E to column 7. With factors and interactions successfully assigned to the available columns, an L_8 is obviously suitable for the design.

Having a total DOF less than or equal to that for the OA is not always a guarantee that a design can be accomplished. Suppose instead of interactions $A \times C$ and $B \times C$ that interactions $A \times C$ and $B \times D$ were to be investigated. The total DOF will still be 7, the same as L_8. By examining Figure A-2, notice that both linear graphs (a) and (b) have a common factor, such as 1, 2, or 4. Because interactions $A \times C$ and $B \times D$ do not have a common factor, an L_8 cannot be used. The next higher-order array should be tried (L_{16} will be needed, as L_{12} is not suitable for interaction).

The experiment designed for *Example 5-6* uses the L_8 OA with column assignments as shown in Table 5-13.

Description of Combinations

The eight trial conditions contained in Table 5-13 can be described individually. Tables 5-14 and 5-15 show trial runs 1 and 2, respectively. The other trial runs can be similarly described. Note that the numbers in the columns where interactions are assigned (columns 3 and 6 in Table 5-13) are not used in the description of trial run 2 (Table 5-15). Normally the interaction column does not need to appear in the description and thus is deleted from the

Table 5-13. L_8 for cake baking experiment—Example 5-6

FACTOR	A	C	A × C	B	D	B × C	E	
COLUMN	1	2	3	4	5	6	7	
TRIAL								RESULT
1	1	1	1	1	1	1	1	66
2	1	1	1	2	2	2	2	75
3	1	2	2	1	1	2	2	54
4	1	2	2	2	2	1	1	62
5	2	1	2	1	2	1	2	52
6	2	1	2	2	1	2	1	82
7	2	2	1	1	2	2	1	52
8	2	2	1	2	1	1	2	78
								Total = 521

description of the trial runs (see trial 2, Table 5-15). Complete design information and analysis for this experiment are shown in Table 5-16.

Running the Experiment

The order in which a specific combination of experiments is run is unaffected by the consideration of the interactions. Conditions 1 through 8 should be done in a random order. A minimum of one trial run per condition must be performed. Repetition of trial runs and the order of repetitions are constrained by time and cost.

Quality Characteristic (Results)

Eight cakes were baked, one for each of the trial runs of Table 5-13. The cakes were then examined by several experienced bakers. Before the cakes were baked, evaluation criteria were established. It was agreed that the cakes were to be evaluated not only for taste but also for appearance and moistness. It was decided that the cakes were to be rated on a scale of 0 to 100, using a scheme to reflect the weighting of each individual attribute of the characteristic. For each condition, the average of the evaluations by the bakers was recorded, as shown in the column marked Results (see Table

Table 5-14. Description of trial 1 (cake baking)—Example 5-6

COLUMN	FACTOR (VARIABLE)	LEVEL	
1	*A*—Eggs	2 eggs	(A_1)
2	*C*—Milk	2 cups	(C_1)
3	$A \times C$ (Eggs × Milk)		
4	*B*—Butter	1 stick	(B_1)
5	*D*—Flour	1 extra scoop	(D_1)
6	$B \times C$ (Butter × Milk)		
7	*E*—Sugar	1 extra scoop	(E_1)

Table 5-15. Description of trial 2 (cake baking)—Example 5-6

COLUMN	FACTOR (VARIABLE)	LEVEL	
1	*A*—Eggs	2 eggs	(A_1)
2	*C*—Milk	2 cups	(C_1)
4	*B*—Butter	1.5 sticks	(B_2)
5	*D*—Flour	2 extra scoops	(D_2)
7	*E*—Sugar	2 extra scoops	(E_2)

5-13). Based on the definition, a higher value of the results was considered favorable. For the purpose of analysis, this constituted the "higher (bigger) is better" type of quality characteristic.

Analysis of Results

The analysis of data including interactions follows the same steps as are taken when there is no interaction. The objectives are the same: (1) determine the optimum condition, (2) identify the individual influence of each factor, and (3) estimate the performance at the optimum condition. The methods for objectives 2 and 3 are the same as before. For the optimum condition, interactions introduce a minor change in the manner in which the optimum levels of factors are identified. To develop a clear understanding of how the optimum condition is selected, the main effects are discussed here in detail. (The details of ANOVA will be covered in Chapter 6, but only the results of a computer analysis will be presented.)

Table 5-16. Analysis of cake baking experiment—Example 5-6

(a) Main effects

COLUMN	FACTOR	DESCRIPTION	LEVEL 1 AVERAGE	LEVEL 2 AVERAGE	DIFFERENCE (2 – 1)
1	*A*	Eggs	64.25	66.00	1.75
2	C	Milk	68.75	61.50	–7.25
3	*A* × C	1 × 2	67.75	62.50	–5.25
4	*B*	Butter	56.00	74.25	18.25
5	*D*	Flour	70.00	60.25	–9.75
6	*B* × C	2 × 4	64.50	65.75	1.25
7	*E*	Sugar	65.50	64.75	–0.75

(b) ANOVA table

COLUMN	FACTOR	DESCRIPTION	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	*A*	Eggs	1	6.125	6.125	5.44	0.49
2	C	Milk	1	105.125	105.125	93.44	10.13
3	*A* × C	Interaction 1 × 2	1	55.125	55.125	49.00	5.26
4	*B*	Butter	1	666.125	666.125	592.11	64.76
5	*D*	Flour	1	190.125	190.125	169.00	18.41
6	*B* × C	Interaction 2 × 4	1	3.125	3.125	2.77	0.19
7	*E*	Sugar	(1)	(1.13)	Pooled		
All others/error			0	1.13	1.13		0.78
Total:			7	1026.880			100.00

(c) Estimate of performance at optimum condition of design/process
Characteristic: Higher (bigger) is better

FACTOR	DESCRIPTION	LEVEL DESCRIPTION	LEVEL NUMBER	CONTRIBUTION
Eggs		3 eggs	2	0.875
Milk		2 cups	1	3.625
Butter		1.5 sticks	2	9.125
Flour		1 extra scoop	1	4.875
Sugar		1 extra scoop	1	0.375
Contribution from all factors (total)				18.875
Current grand average of performance				65.125
Expected result at optimum condition				84.000

The average effect of level 1 of the factor in column 1 (the effect of two eggs) is computed by adding the first four trial results of Table 5-13 and dividing the sum by 4. Note that for trials 1 to 4, factor A (eggs) is assigned level 1 (2 cups). Thus each of these trial runs contains the effect of factor A at level 1 (A_1). The average effect of A_1, therefore, is found by averaging the results of the first four experiments. The notation A_1 with a bar is used for this value. Thus,

$$\overline{A}_1 = (66 + 75 + 54 + 62)/4 = 64.25$$

Similarly, the average effect of level 2 of A is obtained by the last four trial runs because these were runs with factor A at level 2. Hence:

$$\overline{A}_2 = (52 + 82 + 52 + 78)/4 = 66.00$$

Similarly,

$$\overline{C}_1 = 68.75$$

$$\overline{C}_2 = 61.50$$

and,

$$\overline{(A \times C)}_1 = 67.75$$

$$\overline{(A \times C)}_2 = 62.50$$

The calculations for each factor and level are in Table 5-16(a).

The difference between the average value of each factor at levels 2 and 1 indicates the relative influence of the effect. The larger the difference (magnitude), the stronger the influence. The sign of the difference obviously indicates whether the change from level 1 to 2 increases or decreases the result. The main effects are shown visually in Figure 5-4. Figure 5-5 shows the interaction effects of $A \times C$ and $B \times C$.

Ignoring interaction effects for the moment, notice that Table 5-16(a) and Figure 5-4 show an improvement at level 2 only for factors A and B, while level 2 effects for C, D, and E cause a decrease in quality. Hence, the optimum levels for the factors based on the data are A_2, B_2, C_1, D_1, and E_1. Coincidentally, trial 6 tested

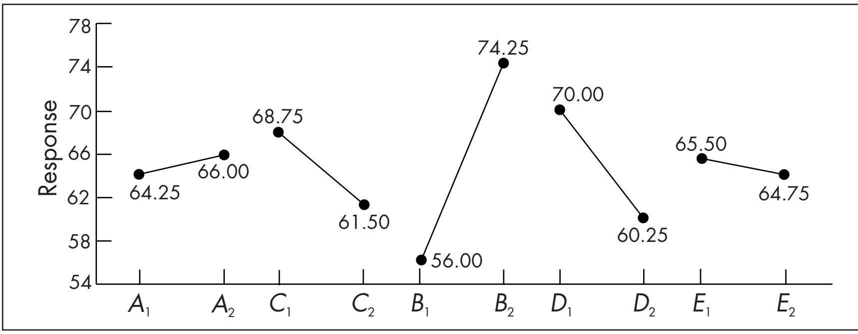


Figure 5-4. Main effects

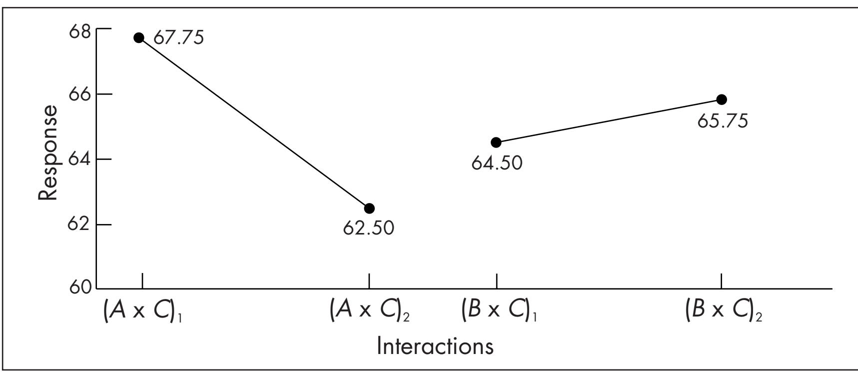


Figure 5-5. Interaction effects

these conditions and produced the highest result (Table 5-13). Because interaction is ignored, the average effects of $(A \times B)_{1,2}$ and $(A \times C)_{1,2}$, shown in Table 5-16(a), are not used in determining the optimum.

Interaction Effects

To determine whether the interaction is present, a proper interpretation of the results is necessary. The general approach is to separate the influence of an interacting member from the influences of the others. In this example, $A \times C$ and $B \times C$ are the interactions with C common to both. The information about C

can be extracted from the columns assigned to factors A, B, and C at the two levels of C. This requires some additional calculations. The steps involved are described below.

The A_1C_1 is first found from the results that contain both A_1 and C_1. Note that A_1C_1 is not the same as the average value in level 1 of Table 5-16(a) for interaction $A \times C$ assigned to column 3 of Table 5-13. This value is $(A \times C)_1$. In this analysis, interaction columns, that is, columns 3 and 6, are not used. Instead, the columns of Table 5-13 that represent the individual factors are used. Examination of column 1 shows that A_1 is contained in rows (trial runs) 1, 2, 3, and 4, but C_1 is in trial runs 1, 2, 5, and 6. Comparing the two, the rows that contain both A_1 and C_1 are 1 and 2. Therefore, A_1C_1 comes from the results of trial runs 1 and 2.

The average effect of $\overline{A_1C_1} = (66 + 75)/2 = 70.50$. The two common trial runs for A_1C_2 are 3 and 4, and the average effect of $\overline{A_1C_2} = (54 + 62)/2 = 58.00$.

In the calculations for $\overline{A_1C_1}$ and $\overline{A_1C_2}$, factor level A_1 is common. The difference between result 70.50 for A_1C_1 and result 58.00 for A_1C_2 is due only to factor C.

Similarly, $\overline{A_2C_1}$, $\overline{A_2C_2}$, $\overline{B_1C_1}$, $\overline{B_1C_2}$, $\overline{B_2C_1}$, and $\overline{B_2C_2}$ are calculated. All of the results are shown below and plotted in Figure 5-6.

$$\overline{A_1C_1} = 70.50 \quad \overline{A_2C_1} = 68.50$$
$$\overline{A_1C_2} = 58.00 \quad \overline{A_2C_2} = 65.00$$
$$\overline{B_1C_1} = 59.00 \quad \overline{B_2C_1} = 78.50$$
$$\overline{B_1C_2} = 53.00 \quad \overline{B_2C_2} = 70.00$$

The intersecting lines on the left represent the presence of interaction between A and C. Of course, for interaction to exist, the lines need to have an angle between them, whether intersecting or not. The parallel lines (representing a lesser angle between) on the right show that B and C probably do not interact. Recall that in Table 5-16(a) the average influence of interaction $(A \times C)_{1,2}$ assigned to column 3 was –5.25.

Further analysis for the significance of this influence is made possible by the ANOVA table in Table 5-16(b), which shows that

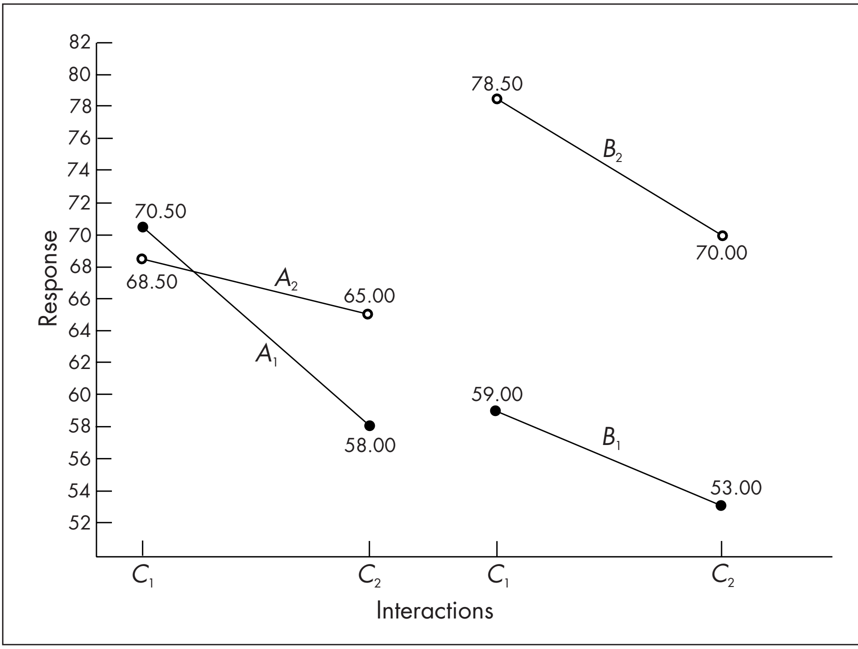


Figure 5-6. Test of interactions

the interaction $A \times C$ (column 3) is 5.26%, compared to the individual main effects of butter (B) 64.76% and flour (D) 18.4%, and so on.

To summarize, the suspected interaction between factors B and C was not observed within the factor ranges studied. The suspected interaction between A and C does exist; its value is 5.26%, based on ANOVA.

To reexamine the optimum condition determined only from the factors A_2, C_1, B_2, D_1, and E_1, we see from Figure 5-6 that $\overline{A_1C_1}$ has a higher value than $\overline{A_2C_1}$. Thus, based on the interaction analysis, the optimum condition must include levels A_1 and C_1. The new optimum conditions become $A_1\,B_2\,C_1\,D_1\,E_1$. However, the performance at the new optimum should be compared with the original optimum before the final determination of the interaction effects.

Consider the initial optimum that excluded the effects of interaction (condition $A_2\, C_1\, B_2\, D_1\, E_1$). Using $\overline{T}$ = average result of eight runs (Table 5-13) = 521/8 = 65.125. Compute the optimum performance using data from Table 5-16(a):

$$
\begin{aligned}
Y_{opt} &= \overline{T} + \left(\overline{A}_2 - \overline{T}\right) + \left(\overline{C}_1 - \overline{T}\right) + \left(\overline{B}_2 - \overline{T}\right) \\
&\quad + \left(\overline{D}_1 - \overline{T}\right) + \left(\overline{E}_1 - \overline{T}\right) \\
&= \overline{T} + \left(66.0 - \overline{T}\right) + \left(68.75 - \overline{T}\right) + \left(74.25 - \overline{T}\right) \\
&\quad + \left(70.00 - \overline{T}\right) + \left(65.50 - \overline{T}\right) \\
&= 65.125 + 0.875 + 3.625 + 9.125 + 4.875 + 0.375 \\
&= 65.125 + 18.875 \\
&= 84.000
\end{aligned}
$$

Similarly, for the revised optimum, considering interaction $A_1\, C_1\, B_2\, D_1\, E_1$, we compute:

(with interaction $A \times C$ only)

$$
\begin{aligned}
Y_{opt} &= \overline{T} + \left(\overline{A}_1 - \overline{T}\right) + \left(\overline{C}_1 - \overline{T}\right) + \left(\left[\overline{A \times C}\right]_1 - \overline{T}\right) \\
&\quad + \left(\overline{B}_2 - \overline{T}\right) + \left(\overline{D}_1 - \overline{T}\right) + \left(\overline{E}_1 - \overline{T}\right) \\
&= \overline{T} + \left(64.25 - \overline{T}\right) + \left(68.75 - \overline{T}\right) + \left(67.75 - \overline{T}\right) + \ldots \\
&= 65.125 + \left(-.875\right) + 3.625 + 2.625 + 9.125 + 4.875 + .375 \\
&= 84.875
\end{aligned}
$$

Y_{opt} can also be calculated by an alternate method as:

$$
\begin{aligned}
Y_{opt} &= \overline{T} + \left(\overline{A_1 C_1} - \overline{T}\right) + \left(\overline{B}_2 - \overline{T}\right) + \left(\overline{D}_1 - \overline{T}\right) + \left(\overline{E}_1 - \overline{T}\right) \\
&= \overline{T} + \left(70.5 - \overline{T}\right) + \left(74.25 - \overline{T}\right) + \left(70 - \overline{T}\right) + \left(65.5 - \overline{T}\right) \\
&= 65.125 + 5.375 + 9.125 + 4.875 + 0.375 \\
&= 84.875
\end{aligned}
$$

Note that when the estimate of performance at the optimum condition includes the interactions between A and C, the net result is obtained from the combined effect of $\overline{A_1C_1}$ in the alternate method and not from using the average value of $A \times C$ from the effect in the third column.

The second calculation yields a value different from the optimum prediction without interaction and should be used to compare the results of the confirmation tests.

As a final check, examine the interaction between B and C. The second pair of lines in Figure 5-6 represents the effect of C at fixed levels of B. The lines are almost parallel, thus indicating little interaction. The ANOVA calculations presented in Table 5-16(b) show a small interaction (0.19%). Observe that the highest value for the pair of lines corresponds to C_1B_2. Comparing C_1B_2 to the revised optimum condition, we find that C_1B_2 is included. Thus, the interaction $B \times C$ has no influence on the optimum. The optimum condition remains as revised for interaction $A \times C$, and no further modification is needed.

Optimum condition = $A_1\, C_1\, B_2\, D_1\, E_1$
Expected performance at optimum condition = 84.875

Key Observations

- In designing experiments with interactions, triangular tables or linear graphs should be used for column assignments. To select the appropriate OA, the types of interactions and their degrees of freedom will have to be considered. The following steps are recommended for the experiment design process:
  1. Select the array based on factors and interactions and their levels. The degrees of freedom of the OA must equal or exceed the DOF of factors and interactions.
  2. Assign factors to the column arbitrarily when no interaction is included. In case interaction is part of the study, treat interacting factors first and reserve columns based on the triangular table to study interaction.
  3. Describe trial conditions by reading across the OA with factors and interactions assigned to the columns.

- For the purpose of analysis, interactions are treated as any other factors; however, their presence is ignored for the preliminary determination of the optimum condition. The relative significance of interactions is obtained from an ANOVA study.
- The determination of the effect of interactions requires a separate study. Such study may suggest a change in the optimum condition.
- When several interactions are included in an experiment, the level selection may become extremely complex.

How should interactions be handled? Should an extra variable be included to study the interaction? If constraints necessitate a choice between including an extra variable or studying an interaction, Taguchi recommends "dig wide, not down." When there is an extra column, study a new variable, not an interaction. On the second pass, if there are strong feelings about the interactions, then they should be included.

More Designs with Interactions

Example 5-7

Design an experiment with five factors at two levels each and two interactions.

Variables: A, B, C, D, E
Interactions: $A \times B$ and $C \times D$

Experiment Design

The five factors and the two interactions each have one DOF. Thus the total DOF is 7. In this case, an L_8 will not work because the triangular table (Tables 5-11 and A-6) shows that there is only one independent triplet in the first seven columns. In other words, if A and B are assigned to columns 1 and 2, column 3 will be reserved for $A \times B$. This means that columns 4, 5, 6, and 7 remain for factors C, D, E, and interaction $C \times D$. However, any combination of these columns 4, 5, 6, and 7 contains only the values 1, 2, or 3; hence, their interaction would involve columns previously assigned to A, B, and $A \times B$. The next higher-order OA is an L_{12}, but it is a special array where interaction effects are distributed, and thus it cannot be used to study interactions. An L_{16} OA is the next higher-order array. Examination of the triangular table (Table A-6) will show that it can be used.

Using Table A-6, arbitrarily assign:

A to column 1	C to column 2
B to column 4	D to column 8

Then $A \times B$ is column 5, $C \times D$ is column 10, and E is column 3.

There were many ways to achieve this column assignment. In this case, using Table A-6, A and B were arbitrarily assigned to columns 1 and 4, with column 5 reserved for interaction $A \times B$. C and D were then assigned to two unused columns such that $C \times D$ becomes a column that is 15 or less and not previously assigned. Columns 2, 8, and 10 were the interacting group of columns selected for factors C, D, and $C \times D$, respectively. The factor E was then assigned to one of the remaining nine columns, column 3. The final design is shown in Table 5-17. Note that columns 6, 7, 9, and 11 to 15 are unassigned but can be used in the analysis for a pooled error estimate.

Example 5-8

Design an experiment with nine factors at two levels each and five interactions, as described below:

Variables: $A, B, C, D, E, F, G, H, I$
Interactions: $A \times B$, $A \times C$, $A \times E$, $A \times F$, and $B \times D$

Experiment Design

The nine factors and five interactions together have 14 DOF. An L_{16} OA has 15 DOF and is a good candidate. Because of the number of interactions, a linear graph of L_{16} is helpful. Because the factor A is common to four of the five interactions, a linear graph with a hub will be used. In Figure A-3, the lower left diagram can be adapted for the design by selecting columns of interest, as shown in Figure 5-7. Start by assigning A to column 1. Then select the ends of four spokes for B, C, E, and F, as shown in Table 5-18. If B is assigned to column 15, then D will be column 8 and $B \times D$ will be column 7. Therefore, when all of the five interactions are assigned to the appropriate columns, the remaining factors can be assigned to the available columns at random.

Table 5-17. L_{16} design with five factors and two interactions—Example 5-7

FACTOR/INTERACTION	A	C	E	B	A × B			D		C × D					
COLUMN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
EXPERIMENT															
1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	0
2	1	1	1	1	1	0	0	2	0	2	0	0	0	0	0
3	1	1	1	2	2	0	0	1	0	1	0	0	0	0	0
4	1	1	1	2	2	0	0	2	0	2	0	0	0	0	0
5	1	2	2	1	1	0	0	1	0	2	0	0	0	0	0
6	1	2	2	1	1	0	0	2	0	1	0	0	0	0	0
7	1	2	2	2	2	0	0	1	0	2	0	0	0	0	0
8	1	2	2	2	2	0	0	2	0	1	0	0	0	0	0
9	2	1	2	1	2	0	0	1	0	1	0	0	0	0	0
10	2	1	2	1	2	0	0	2	0	2	0	0	0	0	0
11	2	1	2	2	1	0	0	1	0	1	0	0	0	0	0
12	2	1	2	2	1	0	0	2	0	2	0	0	0	0	0
13	2	2	1	1	2	0	0	1	0	2	0	0	0	0	0
14	2	2	1	1	2	0	0	2	0	1	0	0	0	0	0
15	2	2	1	2	1	0	0	1	0	2	0	0	0	0	0
16	2	2	1	2	1	0	0	2	0	1	0	0	0	0	0

More examples of experiment designs are described in later chapters and specifically in Chapter 9.

DESIGNS WITH MIXED FACTOR LEVELS

Designs without interactions and with all factors at two levels are of the simpler kind. They are the least cumbersome and most often can be designed by means of the standard OAs. But there are many occasions when more than two levels will have to be included. In the baking process experiment, a third level

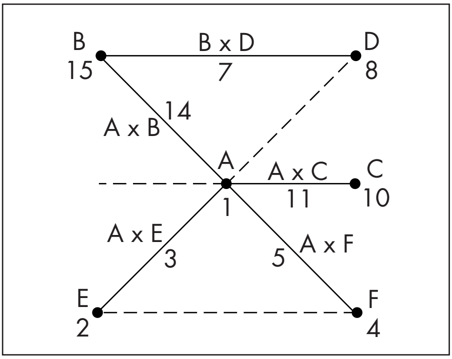


Figure 5-7. Linear graph—Example 5-8

of butter (say, 1.25 sticks, a step between 1 and 1.5) could be specified. The experiment would consist of four factors at two levels and one at three levels. What experiment design is appropriate?

Why are more than two levels needed? For some factors, three levels may be important. Consider an experiment design of a molding process that uses plastic pellet feed stocks from four different vendors; or four different specifications may include a study of materials at four vendor-supplied specifications. The factor (material), in such cases, will have four levels. Another likely reason for more than two levels is that the influence of a factor on the result is suspected to vary nonlinearly. Considering only two levels will give a linear output. Nonlinear behavior can only be determined by a third level, as shown in Figure 5-8.

There are some standard OAs that treat mixed-level factors, but they may not be the most economical or may not even suit one's needs. For most applications involving mixed levels, Taguchi modifies the standard arrays to fit the circumstances. By following his prescription, a two-level column can be upgraded to a four or eight-level column; a four-level column can be upgraded to an eight-level column. On the other hand, a column can also be downgraded by lowering the number of levels, say, from four to three. The method of reducing the levels is done by what is known as *dummy treatment*.

Before considering column modifications, some additional words about DOF are appropriate. Recall that the DOF for a column is its number of levels less 1. Thus, a two-level column has 1 DOF, a three-level column has 2 DOF, and a four-level column has 3 DOF.

Therefore, to create a four-level column, three two-level columns are needed to provide the same DOF. To change one column

Table 5-18. L_{16} design with nine two-level factors and five interactions—Example 5-8

FACTOR/INTERACTION	A	E	A × E	F	A × F	G	B × D	D	H	C	A × C	I		A × B	B
COLUMN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
EXPERIMENT															
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

of an L_8 to a four-level column, three columns are combined. Similarly, to change a column of an L_{16} into an eight-level column, seven of the 15 two-level columns are combined.

Preparation of a Four-Level Column

A four-level column is easily prepared from three two-level columns that are part of an *interacting group of columns*. To demonstrate, consider an L_8. The procedure will also apply for all two-level OAs.

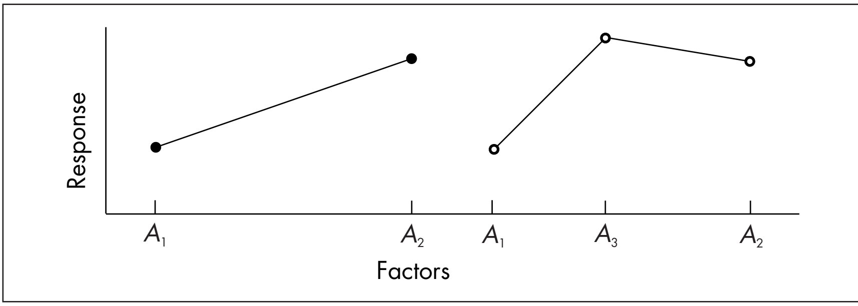


Figure 5-8. Main effects of a factor with two and three levels

Steps

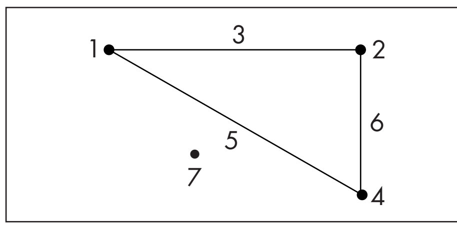


Figure 5-9. Groups of interacting columns for level upgrading

1. From the linear graph for L8, select a set of three interacting columns (Figure 5-9). Example: columns 1, 2, and 3.
2. Select any two columns. Suppose 1 and 2 are selected.
3. Combine the two columns row by row, by following the rules of Table 5-19, to get a combined column such as shown in Table 5-17. Replace the original columns 1, 2, and 3 by the new column that has just been prepared.

Example 5-9

Design an experiment to accommodate one factor at four levels and four others at two levels each.

Variables: A, B, C, D
Interactions: None
Levels: $A = 4$; B, C, $D = 2$

Experiment Design

Factor A has four levels and 3 DOF. The other four two-level factors each have 1 DOF. The total DOF is 7. An L_8 OA, shown in Table 5-20, that has 7 DOF, appears suitable.

Table 5-19. Rules for preparation of a four-level column

LEVEL OF FIRST COLUMN	LEVEL OF SECOND COLUMN	COMBINE TO FORM	LEVEL OF NEW COLUMN
1	1	→	1
1	2	→	2
2	1	→	3
2	2	→	4

Table 5-20. L_8 array—Example 5-9

COLUMN TRIAL	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Building Columns

The first three columns of an L_8 can be combined to produce a four-level column following the procedure previously described.

- Step 1. Start with an original L_8 and select a set of three interacting columns, say 1, 2, and 3.
- Step 2. Ignore column 3 (Table 5-21).
- Step 3. Combine column 1 and 2 into a new column. Follow the procedure as shown by Tables 5-22 and 5-23.
- Step 4. Assign the four-level factor to this new column and the others to the remaining original two-level columns, as shown in Tables 5-24 and 5-25.

The experimental conditions and the subsequent analysis are handled in a manner similarly to the techniques described before.

Table 5-21. L_8 undergoing column upgrade—Example 5-9

COLUMN EXPERIMENT	1	2	3	4	5	6	7
1	1	1		1	1	1	1
2	1	1		2	2	2	2
3	1	2		1	1	2	2
4	1	2		2	2	1	1
5	2	1		1	2	1	2
6	2	1		2	1	2	1
7	2	2		1	2	2	1
8	2	2		2	1	1	2

Table 5-22. Rules for four-level column preparation—Example 5-9

OLD COLUMN			NEW COLUMN
1	1	→	1
1	2	→	2
2	1	→	3
2	2	→	4

Table 5-23. Preparing a four-level column of an L_8 array—Example 5-9

COLUMN EXPERIMENT	1	2	3	4	5	6	7
1	~~1~~	~~1~~ > 1		1	1	1	1
2	~~1~~	~~1~~ > 1		2	2	2	2
3	~~1~~	~~2~~ > 2		1	1	2	2
4	~~1~~	~~2~~ > 2		2	2	1	1
5	~~2~~	~~1~~ > 3		1	2	1	2
6	~~2~~	~~1~~ > 3		2	1	2	1
7	~~2~~	~~2~~ > 4		1	2	2	1
8	~~2~~	~~2~~ > 4		2	1	1	2

Table 5-24. Modified L_8 array with one four-level column—Example 5-9

EXPERIMENT/COLUMN	NEW COLUMN	4	5	6	7
1	1	1	1	1	1
2	1	2	2	2	2
3	2	1	1	2	2
4	2	2	2	1	1
5	3	1	2	1	2
6	3	2	1	2	1
7	4	1	2	2	1
8	4	2	1	1	2

Table 5-25. Modified L_8 with factors assigned (one four-level column)—Example 5-9

FACTOR	*A*	*B*	*C*	*D*	*E*
EXPERIMENT/COLUMN	NEW COLUMN	4	5	6	7
1	1	1	1	1	1
2	1	2	2	2	2
3	2	1	1	2	2
4	2	2	2	1	1
5	3	1	2	1	2
6	3	2	1	2	1
7	4	1	2	2	1
8	4	2	1	1	2

Preparation of an Eight-Level Column

An eight-level column can be prepared by combining a set of seven two-level columns of an L_{16} OA. The procedure is similar to the one used in creating a four-level column. First we need to identify the seven columns involved and then combine the columns using some established guidelines.

Step 1. Select the set of seven columns.

One such set of seven columns consists of columns 1, 2, 3, 4, 5, 6, and 7. The seven columns are an interacting set among three factors, A, B, and C. If A, B, and C are assigned to columns 1, 2, and

4, respectively, the triangular table or the linear graph corresponding to an L_{16} will show that the interaction columns are 3 for $A \times B$, 5 for $A \times C$, 6 for $B \times C$, and 7 for $A \times B \times C$. Here $A \times B \times C$ represents the interaction between factor A (column 1) and interaction $B \times C$ (column 6). The six columns may be represented as a closed triangle in the linear graph and can be selected on this basis. A fourth line connecting the apex and the base represents the interaction $A \times B \times C$, as shown in Figure 5-10.

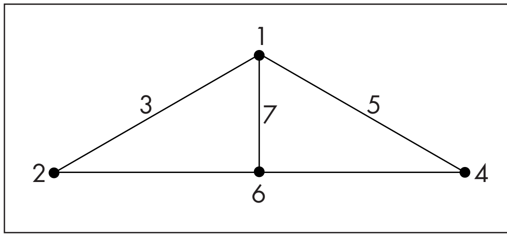


Figure 5-10. Preparation of an eight-level column

Step 2. Select the three columns to be used to form an eight-level column.

Select the three columns where the three factors A, B, and C are assigned. In general, select each apex of the triangle of the linear graph for the set to represent the columns. These are columns 1, 2, and 4 for the set. The remaining four columns are eliminated because the three columns include the four interactions $A \times B$, $A \times C$, $B \times C$, and $A \times B \times C$.

Step 3. Combine three two-level columns into an eight-level column.

Compare numbers in each row of the three columns and combine them using the rules shown in Table 5-26. Note that the rule is not the previous one for the four-level array, although it follows the same pattern.

For the set of columns under consideration, the first, second, and third are columns 1, 2, and 3, respectively (Fig. 5-10). The modified L_{16} array with its upgraded column is shown in Table 5-27. Note that the linear graph (Fig. 5-10) represents seven columns consisting of three main effects and four interactions. Thus, combining the column representing the three main effects includes the four interactions.

Table 5-26. Rules for preparation of an eight-level column for an L_{16} array

COLUMN FIRST	SECOND	THIRD	NEW COLUMN
1	1	1	1
1	1	2	2
1	2	1	3
1	2	2	4
2	1	1	5
2	1	2	6
2	2	1	7
2	2	2	8

DUMMY TREATMENT (COLUMN DEGRADING)

Just as two-level columns of OA can be combined to higher levels, so a higher level column can be decomposed into lower level columns. The method used is known as *dummy treatment*.

Consider an experiment involving four factors, A, B, C, and D, of which A has only two levels, and all of the others have three levels each. The DOF is 7. An L_9 array has four three-level columns with 8 DOF. It could be used if one column can be reduced to the two-level for factor A and the three remaining columns are occupied by factors B, C, and D. In dummy treatment, the third level of $A = A_3$ is formally treated as A_3, as if A_3 exists. But in reality A_3 is set to be either A_1 or A_2.

The design with the modified column (3) of L_9 is shown in Table 5-28. Factor A can be assigned to any column. Note that column 3 was selected such that the modified level $3 = 1'$ occurs once in each group of three trial runs. This distribution enhances the experiment.

Example 5-10

In a casting process used to manufacture engine blocks for a passenger car, nine factors and their levels were identified (Table 5-29). The optimum process parameters for the casting operation

Table 5-27. Converting L_{16} to include an eight-level column

COLUMNS COMBINED TO FORM NEW COLUMNS

NEW COLUMN

COLUMN / EXPERIMENT	1	2		4	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	2	2	1	1	1	1	2	2	2	2
4	1	1	2	2	2	2	2	2	1	1	1	1
5	1	2	3	1	1	1	2	2	1	1	2	2
6	1	2	3	1	2	2	1	1	2	2	1	1
7	1	2	4	2	1	1	2	2	2	2	1	1
8	1	2	4	2	2	2	1	1	1	1	2	2
9	2	1	5	1	1	2	1	2	1	2	1	2
10	2	1	5	1	2	1	2	1	2	1	2	1
11	2	1	6	2	1	2	1	2	2	1	2	1
12	2	1	6	2	2	1	2	1	1	2	1	2
13	2	2	7	1	1	2	2	1	1	2	2	1
14	2	2	7	1	2	1	1	2	2	1	1	2
15	2	2	8	2	1	2	2	1	2	1	1	2
16	2	2	8	2	2	1	1	2	1	2	2	1

are to be determined by experiment. Of the nine factors, two are of three levels each and another of four levels. The remaining six factors are all of two levels each. The DOF is at least 13 if no interactions are considered.

Experiment Design

Because most factors are two-level, a two-level OA may be suitable. Each three-level factor can be accommodated by three columns (modified), and the four-level factor can also be described by three columns, for a subtotal of nine columns. The remaining six two-level factors require one column each. Thus, a minimum of 15 columns is

Table 5-28. Design with degraded column of L_9

FACTOR	B	C	A	D
COLUMN	1	2	3	4
EXPERIMENT				
1	1	1	1	1
2	1	2	2	2
3	1	3	1′	3
4	2	1	2	3
5	2	2	1′	1
6	2	3	1	2
7	3	1	1′	2
8	3	2	1	3
9	3	3	2	1

(′) indicates new modified level
1′ = (level 3)

Table 5-29. Factors of casting process experiment—Example 5-10

FACTOR	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
A: Sand compaction	Plant X	Plant Y	Plant Z	
B: Gating type	Plant X	Plant Y	Plant Z	
C: Metal head	Low	High		
D: Sand supplier	Supplier 1	Supplier 2		
E: Coating type	Type 1	Type 2	Type 3	Type 4
F: Sand permeability	300 perm	400 perm		
G: Metal temperature	1430°F	1460°F		
H: Quench type	450°F	725°F		
I: Gas level	Absent	High amount		

needed. L_{16} satisfies this requirement. Nine columns are to be converted to three four-level columns, then two columns will be reduced by dummy treatment to three-level columns for this experiment.

Normally a three-level column will have 2 DOF. But when it is prepared by reducing a four-level column, it must be counted as 3 DOF because it includes a dummy level. Thus, the total DOF for the experiment is:

6 variables at 2 levels each	...	6 DOF
1 variable at 4 levels each	...	3 DOF
2 variables at 3 levels each	...	6 DOF (dummy treated)
Total DOF	=	15

L_{16} has 15 DOF and therefore is suitable for the design. The three sets of interacting columns used for column upgrading are 1 2 3, 4 8 12, and 7 9 14. The column preparation and assignment follows these steps.

1. Discard column 3 and use columns 1 and 2 to prepare a four-level column, then dummy treat it to a three-level column. Place it as column 1. Assign factor *A* (sand compaction) to this column.
2. Discard column 12 and use columns 4 and 8 to create a four-level column first, then dummy treat it to a three-level column. Call it column 4. Assign factor *B* (gating type) to this column.
3. Discard column 14 and use columns 7 and 9 to create a four-level column for factor *E* (coating type). Call it column 7.
4. Assign the remaining seven two-level factors to the rest of the two-level columns, as shown in Table 5-30(a).

The detail array modified to produce two three-level and one four-level column is shown in Table 5-30(b). Table 5-30(c) shows the modifications to create three four-level columns and the dummy treatment of two columns to three-level columns. Note that in new column 1, the four dummy levels 1′ occur together. In this case, to avoid any undesirable bias due to level 1, the experiment should be carried out by selecting trial conditions in a random order.

Description of Experimental Conditions

Once the factors are assigned, the 16 trial runs are described by the rows of the OA (modified), as shown in Table 5-30(b). With

Table 5-30(a). Casting process optimization design—Example 5-10 (Design variables and their levels)

COLUMN	FACTOR	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Sand compaction	Plant X	Plant Y	Plant Z	
2	(Used with Col 1)	M/U			
3	(Used with Col 1)	M/U			
4	Gating type	Plant X	Plant Y	Plant Z	
5	Metal head	Low	High		
6	Sand supplier	Supplier 1	Supplier 2		
7	Coating type	Type 1	Type 2	Type 3	Type 4
8	(Used with Col 4)	M/U			
9	(Used with Col 7)	M/U			
10	Sand perm	200 perm	300 perm		
11	Metal temperature	1430°F	1460°F		
12	(Used with Col 4)	M/U			
13	Quench type	450°F	725°F		
14	(Used with Col 7)	M/U			
15	Gas level	None	High		

Note: Modified columns 1 2 3, 4 8 12, and 7 9 14.
No interaction.
Objective: Determine process parameter for best casting.
Characteristic: Bigger is better.

experience, the run conditions are easily read from the array. But for the inexperienced, and for large arrays, translating the array notations into actual descriptions of the factor levels may be subject to error. Computer software [7] is available to reduce/eliminate chances of such errors. A printout of the trial conditions for sample trial runs is shown in Table 5-30(d).

Main Effect Plots for Three-Level and Four-Level Factors

The analysis of experimental data follows the same steps as before. The results of a single test run at each of the 16 conditions are shown in Table 5-30(e). The main effects of the factors are presented in Table 5-30(f); the effects for the three- and four-level

Table 5-30(b). Casting process optimization design—Example 5-10

COLUMN / EXPERIMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Expt 1	1	0	0	1	1	1	1	0	0	1	1	0	1	0	1
Expt 2	1	0	0	2	1	1	2	0	0	2	2	0	2	0	2
Expt 3	1	0	0	3	2	2	3	0	0	1	1	0	2	0	2
Expt 4	1	0	0	1	2	2	4	0	0	2	2	0	1	0	1
Expt 5	2	0	0	1	1	1	1	0	0	2	2	0	1	0	2
Expt 6	2	0	0	2	1	1	2	0	0	1	1	0	2	0	1
Expt 7	2	0	0	3	2	1	1	0	0	2	2	0	2	0	1
Expt 8	2	0	0	1	2	1	2	0	0	1	1	0	1	0	2
Expt 9	3	0	0	1	2	1	4	0	0	1	2	0	2	0	2
Expt 10	3	0	0	2	2	1	3	0	0	2	1	0	1	0	1
Expt 11	3	0	0	3	1	2	2	0	0	1	2	0	1	0	1
Expt 12	3	0	0	1	1	2	1	0	0	2	1	0	2	0	2
Expt 13	1	0	0	1	2	2	2	0	0	2	1	0	2	0	1
Expt 14	1	0	0	2	2	2	1	0	0	1	2	0	1	0	2
Expt 15	1	0	0	3	1	1	4	0	0	2	1	0	1	0	2
Expt 16	1	0	0	1	1	1	3	0	0	1	2	0	2	0	1

factors are displayed in Figure 5-11. The optimum combination is easily determined by plotting the main effects of all factors, or from the data of Table 5-30(f) of the main effects by selecting the higher values (because the quality characteristic is "the bigger the better"). Note that for sand compaction the middle level produces the highest value. Such nonlinear behavior of the factor was suspected from previous experience; hence, three levels were selected for the experiment.

COMBINATION DESIGN

Consider an experiment involving three three-level factors and two two-level factors. An experiment design could consider an L_{16}

Table 5-30(c). Casting process optimization design—Example 5-10 (Column upgrading procedure)

1 2 and 3 to form a 3 level col., "New 1"				4 8 and 12 to form a 3 level col., "New 4"				7 9 and 14 to form a 4 level col., "New 7"			
1	2	3	NEW 1	4	8	12	NEW 4	7	9	14	NEW 7
~~1~~	~~1~~	~~1~~	1	~~1~~	~~1~~	~~1~~	1	~~1~~	~~1~~	~~1~~	1
~~1~~	~~1~~	~~1~~	1	~~1~~	~~2~~	~~1~~	2	~~1~~	~~2~~	~~2~~	2
~~1~~	~~1~~	~~1~~	1	~~2~~	~~1~~	~~2~~	3	~~2~~	~~1~~	~~2~~	3
~~1~~	~~1~~	~~1~~	1	~~2~~	~~2~~	~~1~~	4 = 1′	~~2~~	~~2~~	~~1~~	4
~~1~~	~~2~~	~~2~~	2	~~1~~	~~1~~	~~1~~	1	~~2~~	~~1~~	~~2~~	3
~~1~~	~~2~~	~~2~~	2	~~1~~	~~2~~	~~2~~	2	~~2~~	~~2~~	~~1~~	4
~~1~~	~~2~~	~~2~~	2	~~2~~	~~1~~	~~2~~	3	~~1~~	~~1~~	~~1~~	1
~~1~~	~~2~~	~~2~~	2	~~2~~	~~2~~	~~1~~	4 = 1′	~~1~~	~~2~~	~~2~~	2
~~2~~	~~1~~	~~2~~	3	~~1~~	~~1~~	~~1~~	1	~~2~~	~~2~~	~~1~~	4
~~2~~	~~1~~	~~2~~	3	~~1~~	~~2~~	~~2~~	2	~~2~~	~~1~~	~~2~~	3
~~2~~	~~1~~	~~2~~	3	~~2~~	~~1~~	~~2~~	3	~~1~~	~~2~~	~~2~~	2
~~2~~	~~1~~	~~2~~	3	~~2~~	~~2~~	~~1~~	4 = 1′	~~1~~	~~1~~	~~1~~	1
~~2~~	~~2~~	~~1~~	4 = 1′	~~1~~	~~1~~	~~1~~	1	~~1~~	~~2~~	~~2~~	2
~~2~~	~~2~~	~~1~~	4 = 1′	~~1~~	~~2~~	~~1~~	2	~~1~~	~~1~~	~~1~~	1
~~2~~	~~2~~	~~1~~	4 = 1′	~~2~~	~~1~~	~~2~~	3	~~2~~	~~2~~	~~1~~	4
~~2~~	~~2~~	~~1~~	4 = 1′	~~2~~	~~2~~	~~1~~	4 = 1′	~~2~~	~~1~~	~~2~~	3

Note: (′) indicates dummy-treated levels

OA with three columns for each of the three-level factors and two additional columns for the two-level factors. Such a design will utilize 11 of the available 15 columns and require 16 trial runs for the experiment. Alternatively consider the L_9 OA. Three columns satisfy the three-level factors. If the fourth column can be used to accommodate two two-level factors, then L_9 with only nine trial runs could be used.

Indeed, it is possible to combine two two-level factors into a single three-level factor, with some loss of confidence in the results and loss of opportunity to study interactions. The procedure is given below.

Table 5-30(d). Description of individual trial conditions—Example 5-10

TRIAL 1		
Sand compaction M/C	= Plant X	...Level 1
Gating type	= Plant X	...Level 1
Metal head	= Low	...Level 1
Sand supplier	= Supplier 1	...Level 1
Coating type	= Type 1	...Level 1
Sand perm	= 300 perm	...Level 1
Metal temperature	= 1430°F	...Level 1
Quench type	= 450°F	...Level 1
Gas level	= Absent/none	...Level 1
TRIAL 2		
Sand compaction M/C	= Plant X	...Level 1
Gating type	= Plant Y	...Level 2
Metal head	= Low	...Level 1
Sand supplier	= Supplier 1	...Level 1
Coating type	= Type 2	...Level 2
Sand perm	= 400 perm	...Level 2
Metal temperature	= 1460°F	...Level 2
Quench type	= 725°F	...Level 2
Gas level	= High	...Level 2

Define a new factor (XY) to be formed out of the combination of X and Y and assign it to column 4. From the four possible combinations X and Y (X_1Y_1, X_2Y_1, X_1Y_2, and X_2Y_2), select any three and label them as stated below:

X_1Y_1 as $(XY)_1$ that is, level 1 of new factor (XY)
X_2Y_1 as $(XY)_2$ that is, level 2 of new factor (XY)
X_1Y_2 as $(XY)_3$ that is, level 3 of new factor (XY)

Note that one combination, X_2Y_2, is not included. With factor XY assigned, an L_9 OA is shown in Table 5-31. From the array, the trial run conditions defined for trial 1 (row 1) are A_1 B_1 C_1 $(XY)_1$, where $(XY)_1$ is X_1Y_1, which was defined above.

Table 5-30(e). Casting process optimization data—Example 5-10

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVG.
1	67.00						67.00
2	66.00						66.00
3	56.00						56.00
4	67.00						67.00
5	78.00						78.00
6	90.00						90.00
7	68.00						68.00
8	78.00						78.00
9	89.00						89.00
10	78.00						78.00
11	69.00						69.00
12	76.00						76.00
13	78.00						78.00
14	66.00						66.00
15	77.00						77.00
16	87.00						87.00

Table 5-30(f). Casting process optimization design main effects—Example 5-10

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$(L_2 - L_1)$	LEVEL 3	LEVEL 4
1	Sand compaction	70.50	78.50	8.00	78.00	00.00
4	Gating type	77.50	75.00	–2.50	67.50	00.00
5	Metal head	76.25	72.50	–3.75	00.00	00.00
6	Sand supplier	76.25	72.50	–3.75	00.00	00.00
7	Coating type	69.25	72.75	3.50	74.75	80.75
10	Sand permeability	75.25	73.50	–1.75	00.00	00.00
11	Metal temperature	75.00	73.75	–1.25	00.00	00.00
13	Quench type	72.50	76.25	3.75	00.00	00.00
15	Gas level	75.50	73.25	–2.25	00.00	00.00

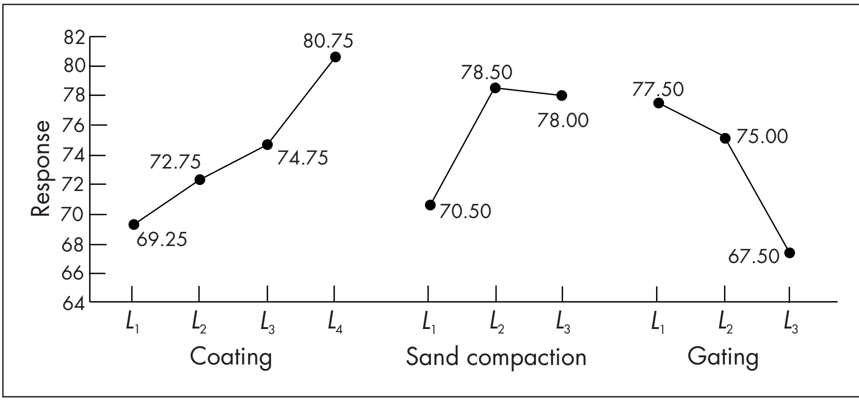


Figure 5-11. Plots of main effects—Example 5-10

Table 5-31. L_9 with five factors

FACTOR	A	B	C	(XY)
COLUMN	1	2	3	4
EXPERIMENT				
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	3
8	3	2	1	3
9	3	3	2	1

The total data are analyzed with the two factors X and Y treated as one, (XY). The analysis yields the main effect of (XY). The individual effect of the constituents X and Y is then obtained as follows:

Main effect of $X = \overline{(XY)}_1 - \overline{(XY)}_2$ and

Main effect of $Y = \overline{(XY)}_1 - \overline{(XY)}_3$

The first equation above, which can be expanded as $(X_1Y_1) - (X_2Y_1)$, shows the effect of X when Y is fixed at Y_1. The second equation, which can be expanded as $(X_1Y_1) - (X_1Y_2)$, shows the effect of Y when X is fixed at X_1.

After determining the main effects, the optimum condition, including the levels of the two factors used in combination design, can be identified. However, the interaction effects between factors X and Y cannot be obtained from the data by this method. Should interaction be important, the experiment design must be based on a larger array such as L_{16}.

DESIGNING EXPERIMENTS TO REDUCE VARIABILITY

Variation is a law of nature. No two things in nature are alike. Examined carefully, man/machine-made parts of a kind also differ. Our goal in quality improvement is to reduce variation. Therefore, variation is our number-one enemy.

Robust products and processes perform consistently on target. To build robustness, we must reduce variability in performance. But what causes variability?

Throughout this text, the terms *factors*, *variables*, and *parameters* synonymously refer to factors that influence the outcome of the product or process under investigation. Taguchi further categorized the factors as controllable factors and noise factors. The factors identified for the baking process experiment, namely, sugar, butter, eggs, milk, and flour, were easily controlled factors. Other factors that are less controllable or too expensive to control, such as oven temperature distribution, humidity, oven temperature cycle band width, and so on, may also influence the optimum product.

Variation in performance occurs mainly due to the influence of control factors and noise factors (uncontrollable). While DOE can identify the influential control factors that can indeed be adjusted to improve the consistency in performance, for most systems the uncontrollable factors are the main cause of variation. In planned DOE, the results vary when a trial condition is repeated. As statistical principles dictate, the more the samples are tested in the same condition the better the information about the variability. Obviously, to capture variability information from

DOE experiments, we must repeat experiments—in other words, have more samples tested in each trial condition.

Because variation is always present in tests with hardware, testing multiple samples is a necessity. The number of samples desired in a study is decided based on expected variability and the cost of the samples. More importantly, Taguchi followed a structure to repeat the test samples, exposing them to the influence of noise, as will be described later in this section.

Before Taguchi, the noise factors were popularly known as nuisance factors. At different times, different methods were followed to deal with them, often trying to control such uncontrollable factors as temperature, humidity, dust in the air, tool wear, and so on. Unfortunately, there were no effective and standard means of dealing with the noise factors. In this regard, Dr. Taguchi offered a revolutionary strategy that caught the attention of the scientific world. His approach has been to not go after the uncontrollable factors, but simply to find other ways to do the job such that the product is shielded from such influence. His strategy for robust design is to reduce variability of the product/process without actually removing the cause of variation. He chooses to leave the noise alone and instead to find suitable levels of the control factors that produce results most immune to noise influence.

In his robust design strategy, Taguchi seeks the desired design not by selecting the best performance under ideal condition but instead by looking for a design that produces consistent performance in the face of uncontrollable factors. To find the factor level most robust to noise influence, Taguchi relies heavily on the interaction between noise and control factors. Interestingly, critiques of the Taguchi methodologies have been pointing to the lack of emphasis on interaction, which many hold as an important part of the analyses.

ROBUST DESIGN STRATEGY

The key to robust design is noise and control factor interactions. When tests are repeated in the same trial condition, the influence of noise factors in the system will cause variation from sample to sample. Because the noise factors are those that are uncontrollable in the production environment, the goal is to select the levels of the controllable factors that produce minimum variation when exposed to the noise factors. Consider the simple example below to understand the concept.

Example 5-11

In a simple study to determine the influence of alcohol consumption during different types of meals on blood alcohol content (BAC), the test parameters were defined as:

Controllable factor—
A: Type of meals
(two levels: A_1 = light snack, A_2 = steak dinner)

Noise factor—
N: Type of alcohol
(two levels: N_1 = hard liquor, N_2 = light beer)

Assume that the individuals under observation have no control over the drinks he/she will be served, but do have control over the meals consumed before going to a party. The effects of alcohol consumption when meals are of the type described are found to be:

$A_1N_1 = 50$, $A_2N_1 = 30$, $A_1N_2 = 25$, and $A_2N_2 = 20$

The numbers shown are the likelihood of exceeding the BAC limit.

From the above data, the control factor and noise effects can be plotted by taking two data points at a time from the set of four data [Fig. 5-12(a) and 5-12(b)]. These plots showing the effects of one factor at various levels of the other are called interaction plots. The angle between the lines, if present, indicates the strength of the presence of interaction. The first of the two plots [Fig. 5-12(a)] indicates that there is interaction between meals and alcohol consumption because the lines are not parallel. The second plot [Fig. 5-12(b)], however, is of most interest. This graph presents the same interaction showing the effects of noise (alcohol) at two levels of the control factor (meals). From the slope of the two lines, it is obvious that the influence of alcohol is much less (shallower line) when a steak dinner is consumed (A_2). So, given an option,

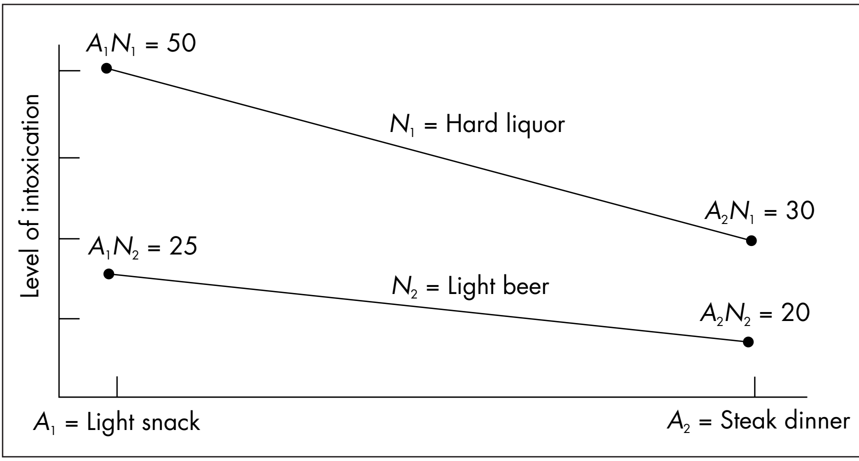


Figure 5-12(a). Type of meal and alcohol interaction plot (factor along x-axis)—Example 5-11

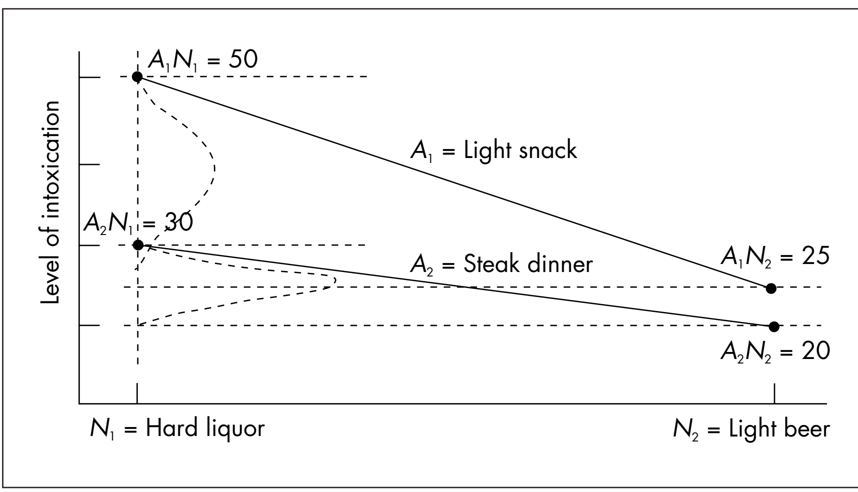


Figure 5-12(b). Alcohol and type of meal interaction plot (noise along x-axis)—Example 5-11 a steak dinner will be more resistant to the influence of alcohol. Obviously, the same concept can be applied to interaction between any control factor and noise in industrial settings.

In the sections that follow, the robust-level selections of multiple factors against single and multiple noises will be described. But because all such experiments will include testing multiple samples in each trial conditions, new ways of analyzing multiple sample results will be covered first.

S/N RATIO—A SMARTER WAY TO ANALYZE MULTIPLE SAMPLE RESULTS

For some experiments, trial runs are easily and inexpensively repeated. For others, repetitions of tests are expensive as well as time consuming. Whenever possible, trials should be repeated, particularly if strong noise factors are present. Repetition offers several advantages. First, the additional trial data confirm the original data points. Second, if noise factors vary during the day, then repeating trials through the day may reveal their influence. Third, additional data can be analyzed for variance around a target value.

When the cost of repetitive trials is low, repetition is highly desirable. When the cost is high or interference with the operation is high, then the number of repetitions should be determined by means of an expected payoff for the added cost. The payoff can be the development of a more robust production procedure or process, or by the introduction of a production process that greatly reduces product variance.

Repetition permits determination of a variance index called the *signal-to-noise (S/N) ratio*. The greater this value, the smaller the product variance around the target value. The signal-to-noise ratio concept has been used in the fields of acoustics, electrical and mechanical vibrations, and other engineering disciplines for many years. Its broader definition and application will be covered in Chapter 6. The basic definition of the S/N ratio is introduced here.

To capture variability, all trial conditions of a planned experiment are repeated such that they have multiple results. A common approach to analyze such results is to use the average of the trial results for the optimum condition. Unfortunately, average alone does not capture complete information about the variability present.

Comparison of multiple data for a part:

8	9	10	=> Avg. = 9
7	9	11	=> Avg. = 9

Comparing the averages of the above two sets, they would look the same. While the distribution of these two sets is different, it will not be captured unless information on standard deviation, range, scatter, and so on, are considered for the comparison purposes. A common indicator for variability is standard deviation (σ). Any scheme that allows comparison of both average and variability is a good measure of population performance.

Mean square deviation (MSD) is one such measure that depends on both average and standard deviation of the data. But MSD requires separate definition for different quality characteristics, as shown below.

S/N is a log (to the base 10) transformation of MSD for the convenience of linearity of influence and a wider range of data.

$$S/N = -10 \log_{10} (\text{MSD})$$

where MSD = mean square deviation from the target value of the quality characteristic.

Consistent with its application in engineering and science, the value of S/N is intended to be large; hence, the value of MSD should be small. Thus, the MSD is defined differently for each of the three quality characteristics considered, smaller, nominal, or larger.

For smaller is better:

$$\text{MSD} = \left(y_1^2 + y_2^2 + y_3^2 + \ldots\right)/n$$

For nominal is best:

$$\text{MSD} = \left(\left(y_1 - m\right)^2 + \left(y_2 - m\right)^2 + \ldots\right)/n$$

Note: It can be shown that MSD in this case equals $\left[(\sigma)^2 + \left(Y_{avg} - m\right)^2\right]$.

For bigger is better:

$$\text{MSD} = \left(1/y_1^2 + 1/y_2^2 + 1/y_3^2 + \ldots\right)/n$$

Table 5-32. L_4 with results and averages

COLUMN TRIAL	1	2	3	R_1	R_2	R_3	AVERAGE
1	1	1	1	5	6	7	6
2	1	2	2	3	4	5	4
3	2	1	2	7	8	9	8
4	2	2	2	4	5	6	5

where y_1, y_2, etc. equal the results of experiments, observations, or quality characteristics such as length, weight, surface finish, and so on.

σ = standard deviation
m = target value of results (above)
n = number of repetitions (y_i)

Consider an experiment with three repetitions, using an L_4 orthogonal array as shown in Table 5-32. In the table, trial 1 is repeated three times, R_1, R_2, and R_3, with results 5, 6, and 7, respectively. The average of these three repetitions is 6. The average is used for the study of the main effects in a manner similar to that described for nonrepeated trials. Slight differences in the analysis of variance for the repetitive case are covered in Chapter 6. For experiments with repetitions, analysis should always use the S/N ratios computed as follows:

Assume that bigger is better is the quality characteristic sought by the experimental data of Table 5-32. Then,

$$\text{MSD} = \left(1/y_1^2 + 1/y_2^2 + 1/y_3^2 + \ldots\right)/n$$

Now, for row 1,

$$y_1^2 = 5 \times 5 = 25 \qquad y_2^2 = 6 \times 6 = 36$$

$$y_3^2 = 7 \times 7 = 49 \qquad n = 3$$

Therefore,

$$\begin{aligned} \text{MSD} &= (1/25 + 1/36 + 1/49)/3 \\ &= (.04 + .02777 + .020408)/3 \\ &= .088185/3 \end{aligned}$$

Table 5-33. L_4 with results and S/N ratios

COLUMN TRIAL	1	2	3	R_1	R_2	R_3	S/N
1	1	1	1	5	6	7	15.316
2	1	2	2	3	4	5	11.47
3	2	1	2	7	8	9	17.92
4	2	2	2	4	5	6	13.62

or

MSD = .029395, a small value.

The S/N ratio is calculated as:

$$\begin{aligned} S/N &= -10\log_{10}(\text{MSD}) \\ &= -10\log_{10}(.029395) \\ &= 15.31 \end{aligned}$$

S/N values for all rows are shown in Table 5-33.

In analysis, the S/N ratio is treated as a single data point at each of the test run conditions. Normal procedure for studies of the main effects will follow. The only difference will be in the selection of the optimum levels. In S/N analysis, the value of MSD or the greatest value of S/N represents a more desirable condition.

TWO-STEP OPTIMIZATIONS

In this approach, product and process designs are achieved by adjusting factor levels to reduce variability. The process follows two distinct steps, with the assumption that reduction of variability is more important than being on the target:

1. Reduce variability by adjusting the levels of factors determined to be influential
2. Adjust performance mean to target by adjusting those factors with less influence on variability

Dr. Taguchi recommends the two-step optimization strategy when multiple factors influence the outcome. The following example demonstrates how robust factor levels are determined when there is only one major noise factor.

The optimization using the above two steps can be achieved by three different independent types of analyses: (a) noise-to-control factor interaction study, (b) mean and standard deviation analysis, and (c) analyses using S/N ratios of results. While all three types give a deeper understanding of the optimization process, the S/N analysis is the recommended approach. The three analyses are explained in the example using the same experimental results.

Example 5-12: Experimental Study to Reduce Rejects Due to Short Shots in an Injection Molding Process

Objective: Reduce rejects due to short shots.

Quality characteristic: Percent of rejects with desirable performance—smaller is better.

Factors and levels: The top six of the list of 18 qualified and "Paretoized" factors were selected for the study. To keep the size of the experiment small and study as many factors as possible, all factors were studied at two extreme ranges of values (two levels). These factors and their levels are shown next as a long list of qualified factors in descending order of importance to the project team.

1. Injection pressure
2. Mold closing speed
3. Mold pressure
4. Backpressure
5. Screw speed
6. Spear temperature
7. Manifold temperature
8. Mold opening speed
9. Mold opening time
10. Forward screw speed
11. Nozzle heater-on time
12. Screw retract speed
13. Cooling time
14. Holding pressure time
15. Ejection speed
16. Coolant type (water/oil)
17. Room temperature (cold/warm)
18. Operator skill level (new/experienced)

Table 5-34. Selected factors and their levels—Example 5-12 (Six among 18 factors selected for the study)

NOTATION	FACTOR DESCRIPTION	LEVEL 1	LEVEL 2
A	Injection pressure	1,800 psi	2,250 psi
B	Mold closing speed	Low (not revealed)	Moderate
C	Mold pressure	600 psi (4.1 MPa)	950 psi
D	Backpressure	950 psi	1,075 psi
E	Screw speed	50 sec.	65 sec.
F	Spear temperature	325°C	380°C
Interaction	Between factors *A* and *B* ($A \times B$), column 3 reserved		

Selected factors and their levels (six among 18 factors selected for the study) are shown in Table 5-34.

Interaction: Interaction between factors A and B ($A \times B$) was identified but not studied.

Noise factors: Among the factors identified, the coolant type was considered uncontrollable, noise factor: Coolant type (water $= N_1$ and oil $= N_2$)

Scope of Experiment: Based on the number of factors (six two-level factors and one interaction), the experiment using an L-8 array and six samples, three in each noise condition, were tested in each trial condition.

The experiment design [Fig. 5-13(a)] shows how the six factors are assigned. The results collected after the tests exposing them to the two noise levels are shown in Figure 5-13(b). The three columns next to the results show the average values under each noise level (N_1 and N_2) and the average of all results in a trial.

(a) Two-Step Optimization Using Noise and Control Factor Interaction

The values of the trial averages [Fig. 5-13(b)] are used to calculate the combined factor and noise (A_1N_1, A_2N_1, and so on) effects and their plots as shown in Figure 5-13(c).

The combined effect of factor A and noise is calculated as:

TRIAL	FACTOR A	B	–	C	D	E	F	RESULTS Noise N_1	Noise N_2
1	1	1	1	1	1	1	1	For each trial condition:	
2	1	1	1	2	2	2	2		
3	1	2	2	1	1	2	2	3 sample results were exposed to noise condition N_1.	
4	1	2	2	2	2	1	1		
5	2	1	2	1	2	1	2		
6	2	1	2	2	1	2	1	3 sample results were exposed to noise condition N_2.	
7	2	2	1	1	2	2	1		
8	2	2	1	2	1	1	2		

Figure 5-13(a). Experimental design for six two-level factors and noise exposure—Example 5-12

	FACTOR							RESULTS (y)						AVERAGE		
TRIAL	A	B	–	C	D	E	F	N_1			N_2			$\overline{N_1}$	$\overline{N_2}$	$\bar{y}$
1	1	1	1	1	1	1	1	11.5	11.8	11.3	14.1	14.5	13.8	11.5	14.3	12.8
2	1	1	1	2	2	2	2	9.2	8.7	8.2	9.3	10.7	9.6	8.7	9.9	9.3
3	1	2	2	1	1	2	2	11.7	11.8	11.5	14.3	14.4	14.1	11.7	14.3	12.9
4	1	2	2	2	2	1	1	12.7	12.7	12.6	15.6	15.6	15.4	12.7	15.5	14.1
5	2	1	2	1	2	1	2	13.8	13.5	13.8	13.3	12.8	12.4	13.7	12.8	13.3
6	2	1	2	2	1	2	1	13.2	13.5	13.4	16.2	16.6	16.4	13.4	16.4	14.9
7	2	2	1	1	2	2	1	12.6	12.9	12.1	15.4	15.8	14.8	12.5	15.3	13.9
8	2	2	1	2	1	1	2	12.3	11.7	12	15.1	14.3	14.2	12.0	14.7	13.3
													Grand averages =>	12.0	14.1	13.1

Figure 5-13(b). Experimental results and calculated trial results averages—Example 5-12

Average effect of A_1N_1 =
(11.5 + 8.70 + 11.7 + 12.70)/4 = 11.15 (Highlighted data)

Average effect of A_2N_1 =
(13.7 + 13.4 + 12.5 + 12.0)/4 = 12.90

Similarly, all other combined effects [Fig. 5-13(c)] are calculated and are used to plot factor effects at each level using N_1 and N_2 along the x-axis. For example, A_1N_1 (11.5) and A_1N_2 (13.49) are

	NOISE AND CONTROL FACTOR INTERACTION EFFECTS											
	A_1	A_2	B_1	B_2	C_1	C_2	D_1	D_2	E_1	E_2	F_1	F_2
Noise N_1	11.15	12.90	11.82	12.21	12.36	11.68	12.14	11.90	12.48	11.57	12.52	11.52
Noise N_2	13.49	14.82	13.31	14.97	14.15	14.13	14.88	13.39	14.30	13.98	15.35	12.93

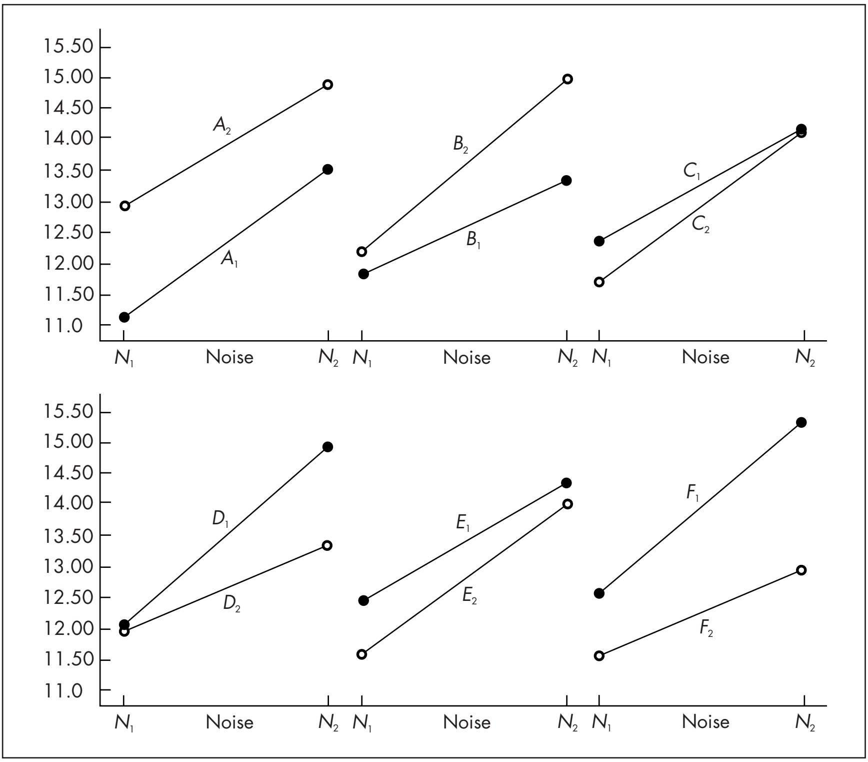


Figure 5-13(c). Calculated noise and control factor interactions and plots ($N\times A$, $N\times B$, ..., $N\times F$)—Example 5-12

used to obtain plot of A_1 line. Plots for all other factor level effects are obtained in the same manner.

Review of the noise and control factor interaction plots [Fig. 5-13(c)] shows that the plots for factors B, C, D, and F have more angle between the lines, indicating that there is significant interaction. Because, for robust design, the line with a shallower angle to horizontal is likely to produce less variation, levels B_1, C_1,

	FACTOR AVERAGE EFFECTS (MAIN EFFECTS)						
	A	*B*	–	*C*	*D*	*E*	*F*
Level 1	12.29	12.57	–	13.25	13.49	13.37	13.94
Level 2	13.84	13.57	–	12.88	12.65	12.78	12.20
Diff. $L_2 - L_1$	1.54	1.00	–	–.37	–.84	–.60	–1.74

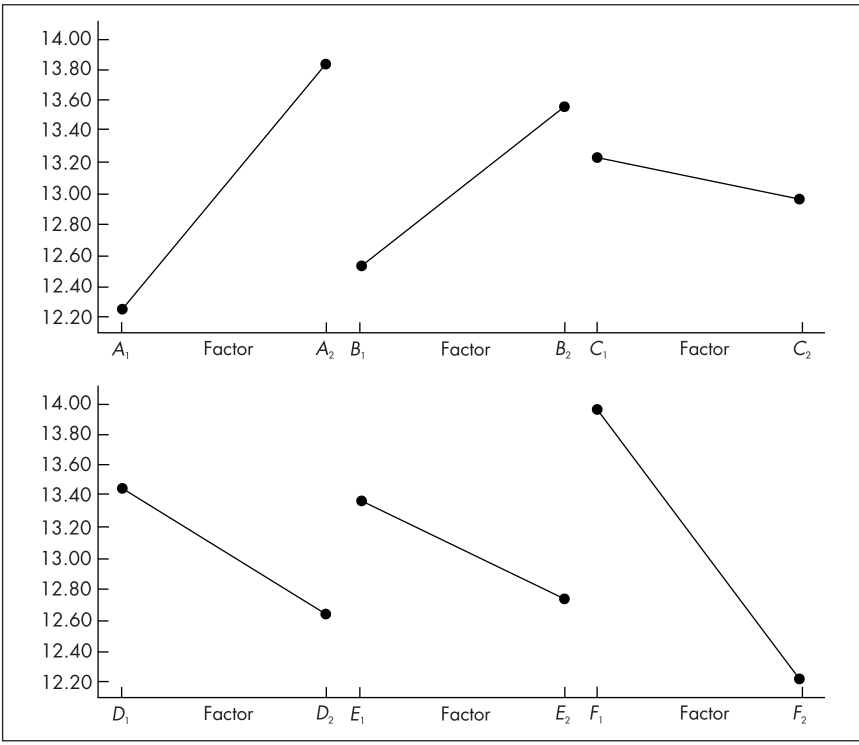


Figure 5-13(d). Calculated factor average effects and plots (Main effects of A, B, C, D, E, and F)

D_2, and F_2 are the choices for these factors. Factors A and E are considered to have interaction of lesser degree and are treated by analysis using the main effects of factors.

Main effects of factor are plotted from the calculated average effects using the trial result averages [last column in Fig. 5-13(b)], as shown in Figure 5-13(d). The levels of the remaining two fac-

TRIAL	FACTOR							RESULTS (y)						AVG $\bar{y}$	SD σ
	A	*B*	–	*C*	*D*	*E*	*F*	N_1			N_2				
1	1	1	1	1	1	1	1	11.5	11.8	11.3	14.1	14.5	13.8	12.8	1.45
2	1	1	1	2	2	2	2	9.2	8.7	8.2	9.3	10.7	9.6	9.3	0.85
3	1	2	2	1	1	2	2	11.7	11.8	11.5	14.3	14.4	14.1	12.9	1.43
4	1	2	2	2	2	1	1	12.7	12.7	12.6	15.6	15.6	15.4	14.1	1.57
5	2	1	2	1	2	1	2	13.8	13.5	13.8	13.3	12.8	12.4	13.3	0.56
6	2	1	2	2	1	2	1	13.2	13.5	13.4	16.2	16.6	16.4	14.9	1.67
7	2	2	1	1	2	2	1	12.6	12.9	12.1	15.4	15.8	14.8	13.9	1.59
8	2	2	1	2	1	1	2	12.3	11.7	12	15.1	14.3	14.2	13.3	1.43
											Grand averages=>			13.1	1.32

Figure 5-13(e). Experimental results and calculated standard deviation of trial results—Example 5-12

tors, A and E, now can be identified from the lower values (QC = smaller is better) of the factor average effects as A_1 and E_2.

Optimization Step Summary

1. Reduce variability by identifying the factors that interact with noise.
   - Factors with strong interaction: B, C, D, F [A and E are found to have less interaction with N; see interaction plot Figure 5-13(c)].
   - Levels for least variability: B_1, C_1, D_2, and F_2
2. Adjust mean by selecting factors with least interaction with noise.
   - Factors with lesser interaction: A and E
   - Levels for mean closer to target: A_1 and E_2 (smaller is better QC, see plots of main effect above)

(b) Two-Step Optimization Using Mean and Standard Deviation

This type of analysis uses standard deviation (σ) of trial results for selecting robust factor levels along with main effects for adjusting mean response. Using the same trial results, standard

	FACTOR EFFECTS ON STANDARD DEVIATION OF RESULTS						
	A	*B*	–	*C*	*D*	*E*	*F*
Level 1	1.33	1.13	–	1.26	1.50	1.26	1.57
Level 2	1.31	1.51	–	1.38	1.14	1.38	1.07
Diff. $L_2 - L_1$	–.01	.37	–	.12	.35	.13	–.50

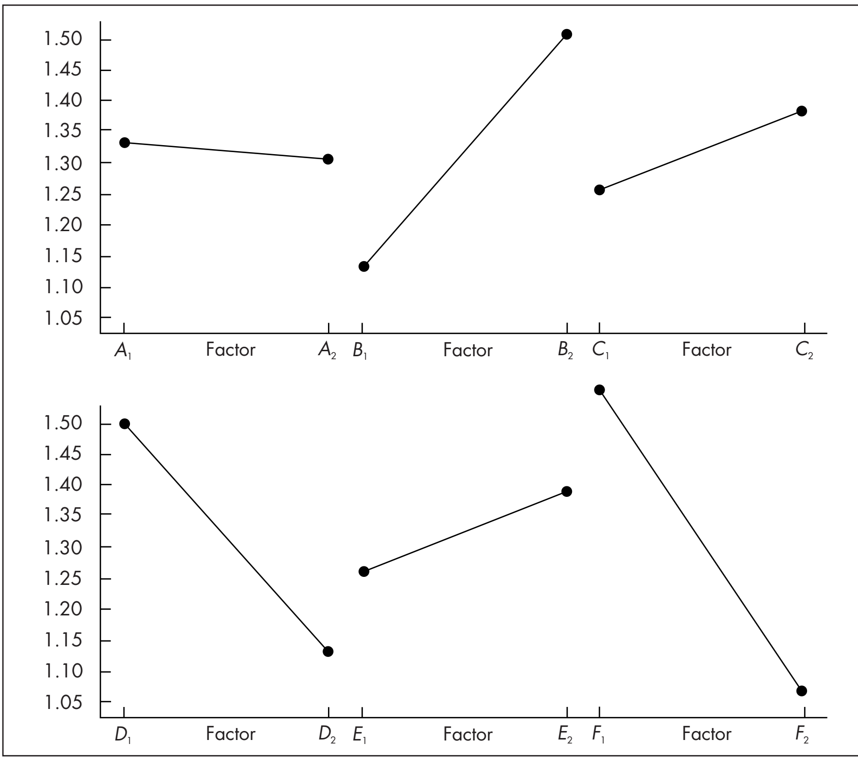


Figure 5-13(f). Calculated average factor effects on standard deviation and plots—Example 5-12

deviation and averages are calculated [Fig. 5-13(e)]. Figure 5-13(f) shows the average effects of factor on σ and the plots. Because variability is never desirable, regardless of the quality characteristic of the process under study, a smaller value (smaller is better) of σ becomes the levels for robust design. Based on the variability [Fig.

								RESULTS (y)						
Tr	A	B	–	C	D	E	F	N_1			N_2			S/N RATIO
1	1	1	1	1	1	1	1	11.5	11.8	11.3	14.1	14.5	13.8	–22.21
2	1	1	1	2	2	2	2	9.2	8.7	8.2	9.3	10.7	9.6	–19.38
3	1	2	2	1	1	2	2	11.7	11.8	11.5	14.3	14.4	14.1	–22.30
4	1	2	2	2	2	1	1	12.7	12.7	12.6	15.6	15.6	15.4	–23.03
5	2	1	2	1	2	1	2	13.8	13.5	13.8	13.3	12.8	12.4	–22.46
6	2	1	2	2	1	2	1	13.2	13.5	13.4	16.2	16.6	16.4	–23.50
7	2	2	1	1	2	2	1	12.6	12.9	12.1	15.4	15.8	14.8	–22.93
8	2	2	1	2	1	1	2	12.3	11.7	12	15.1	14.3	14.2	–22.50
											Grand averages =>			–22.29

Figure 5-13(g). Experimental results and calculated S/N ratios—Example 5-12

5-13(f)], factors B, D, and F are found significant and their levels for robust design are B_1, D_2, and F_2. The levels of the remaining factors are selected based on the main effects as before [A_1, C_2, and E_2 from Fig. 5-13(d)].

Optimization Step Summary

1. Reduce variability by identifying factors with significant effects of standard deviation of results.
   - Significant factors: B, D, and F (A and E are found to have less interaction with N, see interaction plots above)
   - Levels for least variability: B_1, D_2, and F_2
2. Adjust mean by selecting factors with less interaction with noise.
   - Factors with lesser effects on standard deviation: A, C, and E
   - Factor levels: A_1, C_2, and E_2 (smaller is better QC, see plots of main effect above)

(c) Two-Step Optimization Using S/N Ratios

S/N of the trial results [Fig. 5-13(g)], which is directly related to deviation of results from the target, is used for computing fac-

	FACTOR AVERAGE EFFECTS BASED ON S/N RATIOS						
	A	*B*	–	*C*	*D*	*E*	*F*
Level 1	–21.73	–21.89	–	–22.48	–22.63	–22.55	–22.92
Level 2	–22.847	–22.69	–	–22.10	–21.95	–22.03	–21.66
Diff. $L_2 - L_1$	–1.12	–.80	–	.37	.68	.52	1.26

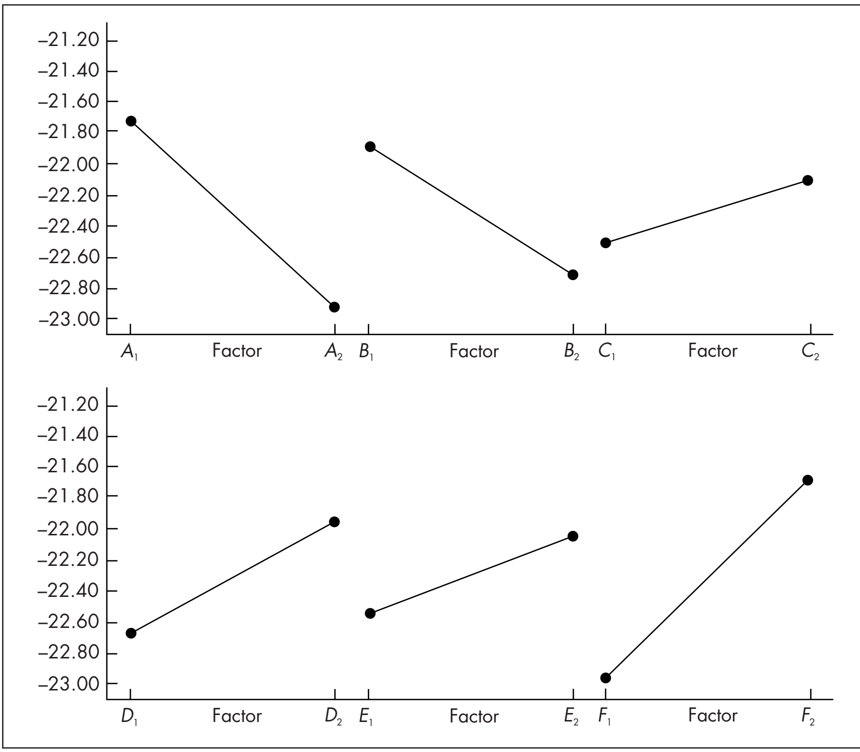


Figure 5-13(h). Plot of average effects of factors (S/N effects of A, B, C, D, E, and F)—Example 5-12

tor average effects as shown in Figure 5-13(h). By definition, no matter the quality characteristic of the original results, larger values of S/N always represent lower variation.

Based on the plot above, the factor levels that shows the highest S/N values are: A_1, B_1, C_2, D_2, E_2, and F_2. All significant factors now can be identified from the rest by performing ANOVA shown

#	FACTOR AND INTERACTION	DOF	SS	V	F	S′	P (%)
1	*A*: Injection pressure	1	2.486	2.486	6.031	2.073	18.96
2	*B*: Mold closing sp.	1	1.277	1.277	3.099	.865	7.91
3	Interaction *A* × *B*	1	2.277	2.277	5.525	1.865	17.05
4	C: Mold pressure	(1)	(.278)	Pooled			
5	*D*: Back pressure	1	.915	.915	2.222	.503	4.60
6	*E*: Screw speed	(1)	(.545)	Pooled			
7	*F*: Spear temperature	1	3.156	3.156	7.657	2.743	25.09
Other/Error		2	.824	.412			26.38
Total		7	10.937				100%

Figure 5-13(i). ANOVA statistics (S/N of results)—Example 5-12

in Figure 5-13(i). Calculation of ANOVA terms and its use will be discussed later (Chapter 6). Our immediate attention is directed to only the last column of ANOVA, which represents the relative percent influence of the factors to the variability of results in statistical and discrete terms.

From ANOVA, factors C and E are found to be insignificant (pooled) and are ignored in the estimation of the expected performance at optimum conditions below. Like in the previous two types of analyses, the levels of factors C and E are determined from the factor average effect plot [Fig. 5-13(d)] using the smaller is better quality characteristic.

Optimum condition: $A_1\ B_1\ D_2\ F_2$ (factors C and E are pooled)

$$
\begin{aligned}
Y_{opt} &= -22.29 + (-21.73 + 22.29) + (-21.89 + 22.29) \\
&\quad + (-21.95 + 22.29) + (-21.66 + 22.29) \\
&= -22.29 + (0.56 + 0.4 + 0.34 + 0.63) \\
&= -22.29 + 1.93 \\
&= -20.36 \text{ S/N}
\end{aligned}
$$

(which translates to 10.4 in the original units of results)

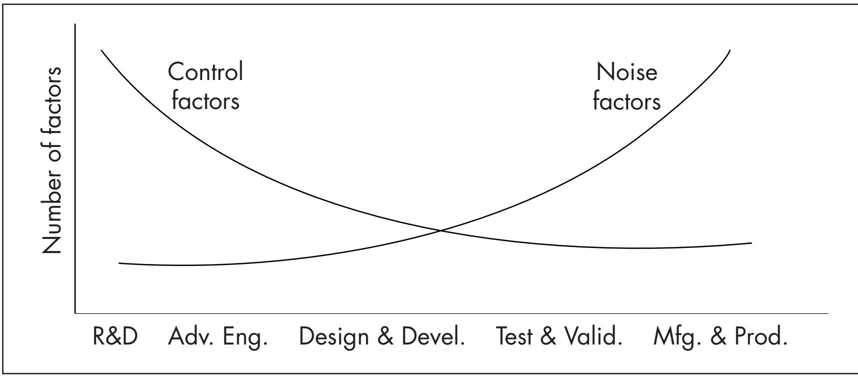


Figure 5-14. Presence of control and noise factors in various stages of product development

Optimization Step Summary

1. Reduce variability by identifying factors with significant effects on S/N.
   - Significant factors: A, B, D, and F
   - Factor levels: A_1, B_1, D_2, and F_2 (larger S/N)
2. Adjust mean by selecting factors with lesser effect on S/N.
   - Factors with lesser effects on S/N: C and E
   - Factor levels: C_2 and E_2 (smaller is better, main effect)

ROBUST DESIGNS AGAINST MULTIPLE NOISES

Repetitions show the variation of the product or process. The variation occurs principally as a result of the uncontrollable factors (noise factors). By expanding the design of the experiment to include noise factors in a controlled manner, optimum conditions insensitive to the influence of the noise factors can be found. These are Taguchi's robust conditions that control production close to the target value despite noise in the production process. Generally, the treatment of noise and control factors varies depending on the experimental studies undertaken during the stages of engineering and production, as depicted in Figure 5-14.

Before describing how the uncontrollable factors are treated, additional definitions are needed:

Controllable factors—Factors whose levels can be specified and controlled during the experiment and in the final design of the product or process.

Noise factors—These are factors that have influence on the product or process results but generally are not maintained at specific levels during the production process or application period.

Inner array—OA of the controllable factors. All experiment designs discussed to this point fall into this category.

Outer array—OA of recognized noise factors. The term outer or inner refers to the usage rather than to the array itself, as will be made clear soon.

Experiment—This refers to the whole experimental process.

Trial condition—Combination of factors/levels at which a trial run is conducted.

Conditions of experiment—Unique combinations of factor levels described by the *inner array* (orthogonal array).

Repetitions or runs—These define the number of observations under the same conditions of an experiment.

The experiment requires a minimum of one run per condition. But one run does not represent the range of possible variability in the results. Repetition of runs enhances the available information in the data. Taguchi suggests guidelines for repetitions.

To incorporate noise factors into the design of the experiment, the factors and their levels are identified in a manner similar to those used for other product and process factors (control factors). For example, if humidity is considered noise, the low and high levels may be considered a factor for the design. After determining the noise factors and their levels for the test, OAs are used to design the conditions of the noise factors that dictate the number of repetitions for the trial runs. The OA used for designing the noise experiment is called an *outer array*.

Assume that three noise factors are identified for the cake baking experiment (Tables 5-12 and 5-16), which utilized an L_8 OA. The noise factors are to be investigated at two levels each. There are four possible combinations of these factors. To obtain complete data, each trial run of L_8 must be repeated for each of the four noise combinations. The noise array selected is an L_4 OA. This outer array,

Table 5-35. Inner and outer orthogonal arrays

Outer array

NOISE FACTORS			
COLUMN / EXPERIMENT	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

Inner array

CONTROL FACTORS								RESULTS			
COLUMN / EXPERIMENT	1	2	3	4	5	6	7	1	2	3	4
1	1	1	1	1	1	1	1				
2	1	1	1	2	2	2	2				
3	1	2	2	1	1	2	2				
4	1	2	2	2	2	1	1				
5	2	1	2	1	2	1	2				
6	2	1	2	2	1	2	1				
7	2	2	1	1	2	2	1				
8	2	2	1	2	1	1	2				

with four combinations of the three noise factors, tests each of the eight trial conditions four times. The experiment design with *inner* and *outer array* is shown by Table 5-35. Note that for the *outer array*, column 3 represents both the third noise factor and the interaction of the first and second noise factor. Note also the arrangement of each array, with the noise (*outer*) array perpendicular to the *inner array*. The complete design is shown by Table 5-36.

For most simple applications, the *outer array* describes the noise conditions for the repetitions. This formal arrangement of the noise factors and the subsequent analysis influences the combination of the controllable factors for the optimum condition. The use of S/N ratio in analysis is strongly recommended.

Table 5-36. Cake baking experiment with noise factors

EXPERIMENT \ COLUMN	1	2	3	4	5	6	7	R_1	R_2	R_3	R_4
1	1	1	1	1	1	1	1				
2	1	1	1	2	2	2	2				
3	1	2	2	1	1	2	2				
4	1	2	2	2	2	1	1				
5	2	1	2	1	2	1	2				
6	2	1	2	2	1	2	1				
7	2	2	1	1	2	2	1				
8	2	2	1	2	1	1	2				

COLUMN	FACTOR DESCRIPTION	LEVEL 1	LEVEL 2
1	Eggs	2 eggs	3 eggs
2	Milk	2 cups	3 cups
3			
4	Butter	1 stick	1.5 sticks
5	Flour	1 extra scoop	2 extra scoops
6			
7	Sugar	1 spoon	2 spoons

	TYPE OF OVEN: 1. Gas 2. Electric	BAKING TIME: 1. +5 min. 2. –5 min.	HUMIDITY 1. 80% 2. 60%
EXPERIMENT \ COLUMN	1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

DESIGN AND ANALYSIS SUMMARY

Application of the Taguchi technique is accomplished in two phases: (1) design of the experiment, which includes determining controllable and noise factors and the levels to be investigated, which determines the number of repetitions, and (2) analysis of the results to determine the best possible factor combination from individual factor influences and interactions. The two activities, experiment design and analysis of test data, are presented (flow charts) in Figures 5-15 and 5-16. The steps involved are briefly described here.

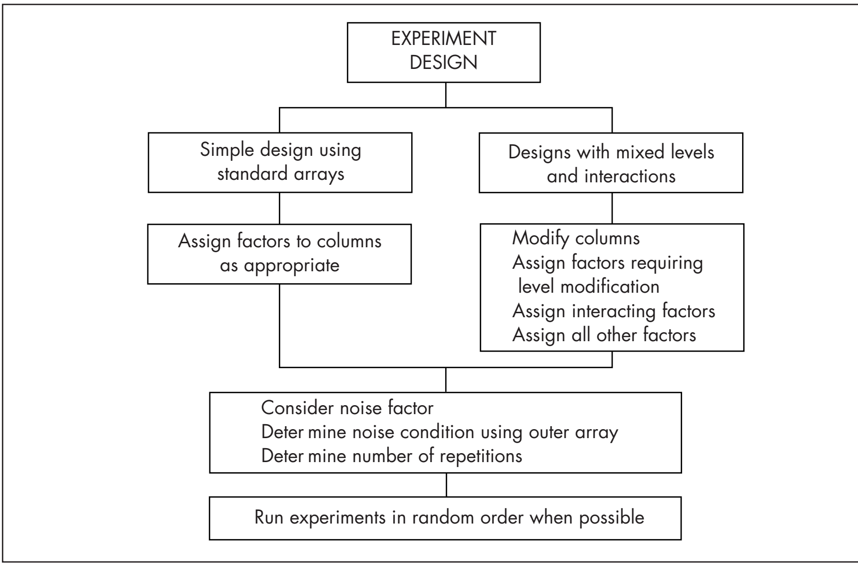


Figure 5-15. Experiment design flow diagram

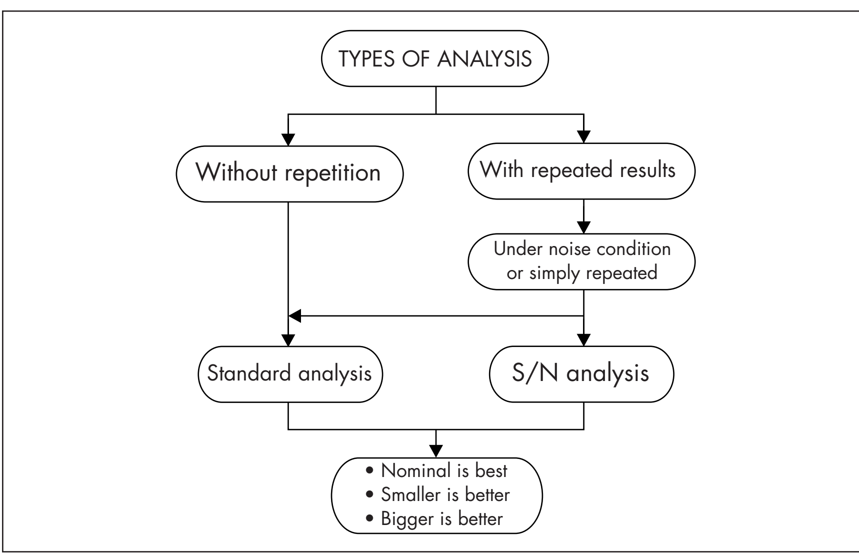


Figure 5-16. Analysis flow diagram

Design of the Experiment

Depending on the factors and levels identified, follow one of the two paths (Fig. 5-15). If all the factors are of the same level, say 2, one of the standard OAs can probably be used. In this case, the factors can be assigned to the columns without much consideration about where they should be placed. On the other hand, if the factors require many levels, or one or more interactions are to be investigated. Then carefully select certain specific columns for factor assignments or level changes. No matter how simple the design, the applicable noise conditions should be identified and a second array (*outer array*) selected to include noise effects. The number of repetitions will be dictated by the number of noise factors. In the absence of a formal layout such as Table 5-35, the number of repetitions will be influenced by time and cost considerations.

Analysis of Results

Analysis of results follows either paths (Fig. 5-16) of repetitions or no repetition. Generally, for a single observation for each trial condition, the standard analysis approach is followed. When there are repetitions of the trial runs, whether by outer array designed noise condition, or under random noise condition, S/N analysis should be performed. The final analysis for the optimum condition is based on one of the three characteristics of quality—greatest, smallest, or nominal.

EXERCISES

5-1. Identify each element (8, 2, 7, and so on) of the notation for an orthogonal array $L_8\,(2^7)$.

5-2. Design an experiment to study four factors, A, B, C, and D, and three interactions, $A \times C$, $C \times D$, and $A \times D$. Select the orthogonal array and identify the columns for the three interactions.

5-3. An experiment with three two-level factors yielded the following results. Determine the average effect of factor C at levels C_1 and C_2.

5-4. Describe the procedure you will follow to design an experiment to study one three-level factor and four two-level factors.
5-5. In an experiment involving piston bearings, an L_8 OA was used in a manner shown in Table 5-37. Determine the description of the trial 7.
5-6. The average effects of the factors involved in Problem 5 are as shown in Table 5-38. If the quality characteristic is "the bigger the better," determine (a) the optimum condition of the design, (b) the grand average of performance, and (c) the performance at the optimum condition. [Ans. (b) 35.01, (c) 38.44]

Table 5-37. Design variables and their levels

COLUMN	FACTOR NAME	LEVEL 1	LEVEL 2
1	Speed	2100 RPM	250 RPM
2	Oil viscosity	At low TP	At high t
3	Interaction 1 × 2	N/A	N/A
4	Clearance	Low	High
5	Pin straightness	Perfect	Bend
6	(Unused)	M/U	
7	(Unused)	M/U	

Table 5-38. Average factor effects

COLUMN	FACTOR NAME	LEVEL 1	LEVEL 2
1	Speed	34.39	35.63
2	Oil viscosity	35.50	34.52
3	Interaction 1 × 2	33.60	36.42
4	Clearance	35.62	34.40
5	Pin straightness	35.31	34.70

6 Analysis of Variance (ANOVA)

THE ROLE OF ANOVA

Taguchi replaces the full factorial experiment with a lean, less expensive, faster, partial factorial experiment. Taguchi's design for the partial factorial is based on specially developed orthogonal arrays (OAs). Because the partial experiment is only a selected set of the full factorial combinations, the analysis of the partial experiment must include an analysis of confidence to qualify the results. Fortunately, there is a standard statistical technique called *analysis of variance* (ANOVA) that is routinely used to provide a measure of confidence. The technique does not directly analyze the data but rather determines the variability (variance) of the data. Confidence is measured from the variance.

Analysis provides the variance of controllable and noise factors. By understanding the source and magnitude of variance, robust operating conditions can be predicted. This is a second benefit of the methodology.

ANOVA TERMS, NOTATIONS, AND DEVELOPMENT

In the analysis of variance, many quantities such as degrees of freedom, sums of squares, mean square, and so on, are computed and organized in a standard tabular format. These quantities and their interrelationships are defined below and their mathematical development is presented.

C.F.	=	correction factor	n	=	number of trials
e	=	error (experimental)	r	=	number of repetitions
F	=	variance ratio	P	=	percent contribution

f	= degrees of freedom	T	= total (of results)
f_e	= degrees of freedom of error	S	= sum of squares
f_T	= total degrees of freedom	S'	= pure sum of squares
		V	= mean square (variance)

Total Number of Trials

In an experiment designed to determine the effect of factor A on response Y, factor A is to be tested at L levels. Assume n_1 repetitions of each trial that includes A_1. Similarly, at level A_2 the trial is to be repeated n_2 times. The total number of trials is the sum of the number of trials at each level, that is,

$$n = n_1 + n_2 + \ldots + n_L$$

Degrees of Freedom (DOF)

DOF is an important and useful concept that is difficult to define. It is a measure of the amount of information that can be uniquely determined from a given set of data. DOF for data concerning a factor equals one less than the number of levels. For a factor A with four levels, A_1 data can be compared with A_2, A_3, and A_4 data but not with itself. Thus, a four-level factor has three DOF. The DOF concept is also applied to columns of OAs as well as the array itself. As with factors, the DOF of a column is its number of levels minus one. Finally, the DOF of an array is the sum of its column DOF. Thus, an L_4 OA with three columns representing two-level factors has three DOF.

The concept of DOF can also be extended to the experiment. An experiment with n trials and r repetitions of each trial has $n \times r$ trial runs. The total DOF becomes:

$$f_T = n \times r - 1$$

Similarly, the DOF for a sum of squares term is equal to the number of terms used to compute the sum of squares, and the DOF of the error term, f_e, is given by:

$$f_e = f_T - f_A - f_B - f_C$$

Sum of Squares

The sum of squares is a measure of the deviation of the experimental data from the mean value of the data. Summing each squared deviation emphasizes the total deviation. Thus,

$$S_T = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$$

where $\bar{Y}$ is the average value of Y_i.

Similarly, the sum of squares of deviations, S_T, from a target value, Y_0, is given by

$$S_T = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2 + n\left(\bar{Y} - Y_0\right)^2 \qquad (6\text{-}1\text{-}1)^*$$

Variance measures the distribution of the data about the mean of the data. Because the data are representative of only a part of all possible data, DOF rather than the number of observations is used in the calculation.

---

*
$$\begin{aligned}
S_T &= \sum_{i=1}^{n} \left(Y_i - Y_0\right)^2 \\
&= \sum_{i=1}^{n} \left(Y_i - \bar{Y} + \bar{Y} - Y_0\right)^2 \\
&= \sum_{i=1}^{n} \left[\left(Y_i - \bar{Y}\right)^2 + 2\left(Y_i - \bar{Y}\right)\left(\bar{Y} - Y_0\right) + \left(\bar{Y} - Y_0\right)^2\right] \\
&= \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2 + \sum_{i=1}^{n} 2\left(Y_i - \bar{Y}\right)\left(\bar{Y} - Y_0\right) + \sum_{i=1}^{n} \left(\bar{Y} - Y_0\right)^2
\end{aligned}$$

because $\sum_{i=1}^{n} \left(Y_i - \bar{Y}\right) = \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} \bar{Y} = n\bar{Y} - n\bar{Y} = 0$

and $\sum_{i=1}^{n} \left(\bar{Y} - Y_0\right)^2 = n\left(\bar{Y} - Y_0\right)^2$

The above equation becomes $S_T = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2 + n\left(\bar{Y} - Y_0\right)^2$.

$$\text{Variance} = \frac{\text{Sum of squares}}{\text{Degrees of freedom}}$$

or $$V = S_T/f$$

When the average sum of squares is calculated about the mean, it is called the general variance. The general variance, σ^2, is defined as:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2 \tag{6-1-2}$$

Let m represent the deviation of the mean, $\bar{Y}$, from the target value, Y_0, that is,

$$m = \left(\bar{Y} - Y_0\right)^2 \tag{6-1-3}$$

Substituting Eqs. (6-1-2) and (6-1-3) into Eq. (6-1-1),

$$S_T = n\sigma^2 + nm^2 = n\left(\sigma^2 + m^2\right) \tag{6-1-4}$$

Thus the total sum of squares of deviations (S_T) from the target value (Y_0) is the sum of the variance about the mean and the square of the deviation of the mean from the target value multiplied by the total number of observations made in the experiment.

S_T of Eq. (6-1-4) also represents the expected statistical value of S_T. In this book, rigorous proofs are omitted unless necessary to clarify an idea or concept. Further, the symbol S_T is used for both the expected value and the computed value for a given sample.

The total sum of squares, S_T (Eq. 6-1-4), gives an estimate of the sum of the variations of the individual observations about the mean, $\bar{Y}$, of the experimental data and the variation of the mean about the target value, Y_0. This information is valuable for controlling manufacturing processes, as the corrective actions to reduce the variations around the mean, $\bar{Y}$, that is, to reduce σ^2, are usually not identical to those actions that move the mean toward the target value. When the total sum of squares, S_T, is separated into its constituents, the variation can be understood and an appropriate strategy to bring the process under control can be easily developed. Furthermore, the information thus acquired can be effectively utilized in statistical process control (SPC).

Mean Sum (of Deviations) Squared

Let $T = \sum_{i=1}^{n}(Y_i - Y_0)$ be the sum of all deviations from the target value. Then, the mean sum of squares of the deviation is:

$$S = T^2/n = \left[\sum_{i=1}^{n}(Y_i - Y_0)\right]^2 \Big/ n \tag{6-2}$$

Eq. (6-2) can thus be written as:

$$S_m = nm^2 \, *$$

It is important to note that even though from an oversimplistic derivation of the value of $S_m = nm^2$, its statistical estimate or the expected value includes one part of the general variance. Therefore, representing the statistically expected value by $E(S_m)$:

$$E(S_m) = S_m = \sigma^2 + nm^2 \tag{6-3}$$

The term $(S_T - S_m)$ is usually referred to as the error sum of squares and can be obtained from Eqs. (6-1-4) and (6-3).

Therefore,

$$S_e = S_T - S_m = (n-1)\sigma^2$$

Rewriting the equation, $S_T = S_e + S_m$. Thus, the total effect of variance, S_T, can be decomposed into the mean deviation, S_m, and the deviation, S_e, about the mean. Thus, individual effects can be analyzed. Let

---

* $$S_m = \frac{1}{n}\left[(Y_1 - Y_0) + \cdots + (Y_n + Y_0)\right]^2$$

which can also be expressed as:

$$S_m = \frac{1}{n}\left[(Y_1 + Y_2 + \cdots + Y_n - nY_0)\right]^2$$

or $$S_m = \frac{1}{n}\left[(n\bar{Y} - nY_0)\right]^2$$

or $$S_m = \frac{n^2}{n}\left[(\bar{Y} - Y_0)\right]^2$$

$$S_m = nm^2$$

$$Y_1 - Y_0 = 3 \qquad Y_4 - Y_0 = 4$$
$$Y_2 - Y_0 = 5 \qquad Y_5 - Y_0 = 6$$
$$Y_3 - Y_0 = 7 \qquad Y_6 - Y_0 = 8$$

where Y_0 is a target value, then

$$\begin{aligned} S_T &= 3^2 + 5^2 + 7^2 + 4^2 + 6^2 + 8^2 \\ &= 199 \end{aligned}$$

$$\begin{aligned} S_m &= (3+5+7+4+6+8)^2/6 \\ &= 33^2/6 \\ &= 181.5 \end{aligned}$$

$$\begin{aligned} \bar{Y} &= (3+5+7+4+6+8)/6 \\ &= 5.5 \end{aligned}$$

and $$\begin{aligned} S_e &= \left[(3-\bar{Y})^2 + (5-\bar{Y})^2 + \ldots\right] \\ &= \left[(3-5.5)^2 + (5-5.5)^2 + \ldots(8-5.5)^2\right] \\ &= 17.5 \end{aligned}$$

Note that $S_e = S_T - S_m = 199 - 181.5 = 17.5$

Also, because the standard deviation of the data, 3, 5, 7, 4, 6, and 8, is equal to 1.8708,

$$\begin{aligned} S_e &= (n-1)\sigma^2 \\ &= (6-1)\times(1.8708)^2 \\ &= 17.5 \end{aligned}$$

Degrees of Freedom Sums

The DOF f_e, f_T, and f_m of the sums of squares S_e, S_T, and S_m are as follows:

$f_T = n =$ number of data points

$f_m = 1$ (always for the mean)

$f_e = f_T - f_m = (n - 1)$

As pointed out earlier, the DOF f_T is equal to n because there are n independent values of $(Y_i - Y_0)^2$. For investigating the effect of factors at different levels, the DOF is usually one less than the number of observations.

To summarize:

$$S_T = n\sigma^2 + nm^2 \tag{6-4}$$

$$S_m = \sigma^2 + nm^2 \tag{6-5}$$

$$S_e = S_T - S_m = (n-1)\sigma^2 \tag{6-6}$$

Also, as stated earlier, variance V is

$$V = S/f$$

Therefore,

$$\begin{aligned} V_T &= S_T/f_T = \sigma^2 + m^2 && \text{(total variance)} \\ V_m &= S_m/f_m = \sigma^2 + nm^2 && \text{(mean variance)} \\ V_e &= (S_T - S_m)/f_e = \sigma^2 && \text{(error variance)} \end{aligned}$$

The example that follows should clarify the application of the concepts developed above. The data for this example are fictitious but suffice for the purpose of illustrating the principles.

ONE-WAY ANOVA

One-Factor One-Level Experiment

When one-dimensional experimental data (one response variable) are analyzed using ANOVA, the procedure is termed a one-way analysis of variance. The following problem is an example of a one-way ANOVA. Later, ANOVA will be extended to multidimensional problems.

Example 6-1

To obtain the most desirable iron castings for an engine block, a design engineer wants to maintain the material hardness at 200 BHN. To measure the quality of the castings being supplied by the foundry, the hardness of 10 castings chosen at random from a lot is measured, as displayed in Table 6-1.

Table 6-1. Hardness of cylinder block castings—Example 6-1

SAMPLE	HARDNESS	SAMPLE	HARDNESS
1	240	6	180
2	190	7	195
3	210	8	205
4	230	9	215
5	220	10	215

The analysis:

$$f_T = \text{total number of results} - 1$$
$$= 10 - 1 = 9$$
$$Y_0 = \text{desired value} = 200$$

the mean value is:

$$\bar{Y} = \begin{pmatrix} 240+190+210+230+220+ \\ 180+195+205+215+215 \end{pmatrix} \Big/ 10$$
$$= 210$$

then $$S_T = (240-200)^2 + (190-200)^2 + (210-200)^2 + (230-200)^2 + (220-200)^2 + (180-200)^2 + (195-200)^2 + (205-200)^2 + (215-200)^2 + (215-200)^2$$
$$= 4000$$

and $$S_m = n\left(\bar{Y} - Y_0\right)^2 = 10(210-200)^2 = 1000$$
$$S_e = S_T - S_m = 4000 - 1000 = 3000$$

And the variance is calculated as follows:

$$V_T = S_T / f_T = 4000/9 = 444.44$$
$$V_m = 1000/1 = 1000$$
$$V_e = (S_T - S_m)/f_e = (4000 - 1000)/9 = 333.33$$

These results are summarized in Table 6-2. Table 6-3 represents a generalized format of the ANOVA table.

Table 6-2. Analysis of variance (ANOVA) table—Example 6-1

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
Mean (m)	1	1000	1000.00			
Error (e)	9	3000	333.33			
Total	10	4000				

Table 6-3. Generalized ANOVA table for randomized one-factor designs

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
Mean (m)	f_m	S_m	S_m / f_m			
Error (e)	f_e	S_e	S_e / f_e			
Total	f_T	S_T				

The data cannot be analyzed further, but analysis of the variance of the data can provide additional information about the data.

Let F be the ratio of total variance to the error variance. F coupled with the degrees of freedom for V_T and V_e provides a measure for the confidence in the results.

To complete the analysis, the error variance V_e is removed from S_m and added to S_e. The new values are renamed as

S'_m = pure sum of squares

S'_e = pure error

This reformulation allows calculation of the percent contribution, P, for the mean, P_m, or for any individual factor (P_A, P_B, and so on).

Table 6-3 presents the complete format for analysis F, S, and P. These parameters are described below in greater detail.

Variance Ratio

The variance ratio, commonly called the F statistic, is the ratio of variance due to the effect of a factor and variance due to the error term. (The F statistic is named after Sir Ronald A. Fisher.) This ratio is used to measure the significance of the factor under investigation with respect to the variance of all of the factors included in the error term. The F value obtained in the analysis is compared with a value from standard F-tables for a given statistical level of significance. The tables for various significance levels and different degrees of freedom are available in most handbooks of statistics. Tables B-1 through B-5 in Appendix B provide a brief list of F factors for several levels of significance.

To use the tables, enter the DOF of the numerator to determine the column and the DOF of the denominator to determine the row. The intersection is the F value. For example, the value of $F_{.10}(5, 30)$ from the table is 2.0492, where 5 and 30 are the DOF of the numerator and denominator, respectively. When the computed F value is less than the value determined from the F-tables at the selected level of significance, the factor does not contribute to the sum of squares within the confidence level. Computer software, such as [7], simplifies and speeds the determination of the level of significance of the computed F values.

The F values are calculated by:

$$F_m = V_m / V_e$$

$$F_e = V_e / V_e = 1 \tag{6-7}$$

and for a factor A it is given by:

$$F_A = V_A / V_e \tag{6-8}$$

Pure Sum of Squares

In Eqs. (6-4), (6-5), and (6-6), for each sum of squares there is a general variance term, σ^2, expressed as DOF $\times$ σ^2.

When this term is subtracted from the sum of squares expression, the remainder is called the pure sum of squares. Because S_m has only one DOF, it therefore contains only one σ^2, that is, V_e. Thus, the pure sum of squares for S_m is:

$$S'_m = S_m - V_e = \sigma^2 + nm^2 - \sigma^2 = nm^2$$

The portion of error variance subtracted from the sum of squares for S_m is added to the error term. Therefore,

$$S'_e = S_e + V_e \tag{6-9}$$

If factors A, B, and C, having DOF f_A, f_B, and f_C, are included in an experiment, their pure sum of squares are determined by:

$$S'_A = S_A - f_A \times V_e \tag{6-10}$$
$$S'_B = S_B - f_B \times V_e$$
$$S'_C = S_C - f_C \times V_e$$
$$S'_e = S_e + (f_A + f_B + f_C) \times V_e \tag{6-11}$$

Percent Contribution

The percent contribution for any factor is obtained by dividing the pure sum of squares for that factor by S_T and multiplying the result by 100. The percent contribution is denoted by P and can be calculated using the following equations:

$$P_m = S'_m \times 100/S_T$$
$$P_A = S'_A \times 100/S_T$$
$$P_B = S'_B \times 100/S_T$$
$$P_C = S'_C \times 100/S_T$$
$$P_e = S'_e \times 100/S_T \tag{6-12}$$

The ANOVA Table 6-2 can now be completed as follows. Using Eqs. (6-7) and (6-8) gives

$$F_m = V_m/V_e = 1000/333.33 = 3.00$$
$$F_e = V_e/V_e = 333.33/333.33 = 1.00$$

The pure sum of squares obtained using Eqs. (6-9) and (6-10) is shown below:

Table 6-4. Completed ANOVA table—Example 6-1

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
Mean (m)	1	1000	1000.00	3.00	666.67	16.67
Error (e)	9	3000	333.33	1.00	3333.33	83.33
Total	10	4000				100.00

Table 6-5. Completed generalized ANOVA table for randomized one-factor design

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, $P/100$
Mean (m)	f_m	S_m	$V_m = S_m/f_m$	V_m/V_e	$S_m - V_e$	S'_m / S_T
Error (e)	f_e	S_e	$V_e = S_e/f_e$	—	$S_e + V_e$	S'_e / S_T
Total	f_T	S_T				

$$S'_m = S_m - V_e = 1000 - 333.33 = 666.67$$
$$S'_e = S_e + V_e = 3000 + 333.33 = 3333.33$$

And the percent contribution is calculated by using Eqs. (6-11) and (6-12):

$$P_m = S'_m \times 100/S_T = 666.67/4000 = 16.67$$
$$P_e = S'_e \times 100/S_T = 3333.33/4000 = 83.33$$

The completed ANOVA tables are shown in Table 6-4.

A generalized ANOVA table for one-factor randomized design is shown in Table 6-5.

Returning to Table 6-3, the computed value for F_m, 3.00, is less than the value from Table C-1 for $F_{.1}$ (1,9), that is, 3.3603. Hence, with 90% confidence (10% risk), the castings appear to be similar. The apparent data spread contributes only 16.67% to the sample variability (sum of squares) whereas the remaining 83.33% variation is caused by other factors.

ONE-FACTOR TWO-LEVEL EXPERIMENTS (ONE-WAY ANOVA)

Example 6-2

In *Example 6-1*, an experiment with one response variable and one factor at one level was considered, the factor being the source of the cylinder blocks. Now consider the case with two different vendors supplying the castings. These two sources are assumed to use similar casting processes. Therefore, a new experiment is described with one factor, hardness of castings, from two sources, A_1 and A_2. The question to be resolved is whether the castings being supplied by the two vendors are statistically of the same quality. If not, which one is preferable? The target hardness, 200 BHN, is unchanged.

Ten samples from each of the two castings sources were drawn at random and their hardness was measured. The test yielded the results shown in Table 6-6.

The analysis of this test proceeds as for the experiment of *Example 6-1*. Note that the error sum of squares term, S_e, as given in Eq. (6-11), contains the variation of the mean and that of factor A. Therefore, to separate the effect of vendors, that is, factor A, the sum of squares term, S_A, must be isolated from S_e. The sum of squares for factor A can be calculated by:

$$S_A = \sum_{k=1}^{L} \frac{1}{n_k} \left[\sum_{i=1}^{n} \left(A_{ik} - Y_0 \right) \right]^2 - \frac{T^2}{n} \tag{6-13}$$

where

L = number of levels

n_i, n_k = number of test samples at levels A_i and A_k, respectively

Table 6-6. Measured hardness—Example 6-2

HARDNESS OF CASTINGS FROM VENDOR A_1			HARDNESS OF CASTINGS FROM VENDOR A_2		
240	180	190	197	202	198
195	210	205	205	203	192
230	215	220	208	199	195
215			201		

T = sum total of all deviations from the target value
n = total number of observations = $n_1 + n_2 + \ldots + n_j$

The T^2/n in Eq. (6-13) is a term similar to S_m and is called the correction factor, C.F.

The expression for the total sum of squares can now be written as:

$$S_T = S_e + S_m + S_A \tag{6-14*}$$

The DOF equation will be:

$$f_T = f_e + f_A$$

The analysis:

$$Y_0 = 200\,(\text{unchanged from } \textit{Example 6-1})$$

$$\begin{aligned} S_T &= (240-200)^2 + (180-200)^2 + \ldots + (215-200)^2 \\ &\quad + (197-200)^2 + \ldots + (195-200)^2 \\ &\quad + (201-200)^2 - \text{C.F.} \\ &= 4206 - 500 \\ &= 3706 \end{aligned}$$

As $T^2/n = \text{C.F.}$

$$\begin{aligned} &= \left[(Y_1 - Y_0) + (Y_2 - Y_0) + \ldots\right]^2 / n \\ &= \begin{bmatrix} (40 - 20 + \ldots - 15 + 15) \\ +(-3 + 2 + \ldots - 5 + 1) \end{bmatrix}^2 \Big/ 20 \\ &= 500 \end{aligned}$$

Using Eq. (6-13), the value of S_A, the square sum for the effect of factor A (vendors), is obtained as:

---

* Taguchi considers deviation from the target more significant than that about the mean. The cost of quality is measured as a function of the deviations from the target. Therefore, Taguchi eliminates the variation about the mean from Eq. (6-14) by redefining S_T as follows:

$$S_T = \sum_{i=1}^{n} (Y_i - Y_0)^2 - \text{C.F.} = S_e + S_A$$

or $S_e = S_T - S_A$

$$\begin{aligned} S_A = {} & \text{squares of sum for vendor } A_1 \\ & + \text{squares of sum for vendor } A_2 \\ & - \text{C.F.} \end{aligned}$$

or $$S_A = \left[\left(Y_{iA_1} - Y_0\right)\right]^2 \Big/ n_{A_1} + \left[\left(Y_{iA_2} - Y_0\right)\right]^2 \Big/ n_{A_2} + T^2/n$$

$$= \frac{\left[(240-200)+\cdots+(215-200)\right]^2}{10}$$

$$+ \frac{\left[(197-200)+\cdots+(201-200)\right]^2}{10} - 500$$

$$= 1000 + 0 - 500$$

$$= 500$$

Also $$\begin{aligned} S_e &= S_T - S_A \\ &= 3706 - 500 \\ &= 3206 \end{aligned}$$

and the DOF will be:

$$\begin{aligned} f_T &= 20 - 1 = 19 \\ f_A &= 2 - 1 = 1 \\ f_e &= 19 - 1 = 18 \end{aligned}$$

Also, $V_A = S_A/f_A = 500/1 = 500$

and $V_e = S_e/f_e = 3206/18 = 178.11$

The variance ratio, F, will therefore be as shown below:

$$\begin{aligned} F_A &= V_A/V_e = 500/178.11 = 2.81 \\ F_e &= V_e/V_e = 1 \end{aligned}$$

To complete the ANOVA table, the pure sum of squares and percentage contributions are needed. The pure sum of squares is computed using Eq. (6-10) whereas the percentage contribution is calculated using Eq. (6-12).

The pure sum of squares are:

$$S'_A = S_A - f_A \times V_e = S_A - 1 \times V_e = 500 - 178.11 = 321.87$$

$$S'_e = S_e + f_A \times V_e = S_e - 1 \times V_e = 3206 + 178.11 = 3384.11$$

and the percentage contributions are given by:

$$P_A = S'_A \times 100/S_T = 321.89 \times 100/3706 = 8.68$$

$$P_e = S'_e \times 100/S_T = 3384.11 \times 100/3706 = 91.31$$

The complete ANOVA table is shown in Table 6-7.

Because the degree of freedom for the numerator is 1 and that for the denominator is 18, from the F-tables at 0.10 level of significance (90% confidence) we obtain $F_{.1}$ (1,18) = 3.007. Because the computed value of the F factor is smaller than the limiting values obtained from the table, no significant difference between the two sources of the castings can be concluded. Of the observed variation, only 8.68% is due to the vendor and 91.32% is due to the error and other factors not included in the study. The range of the hardness data of Table 6-6 is 180 to 240 from vendor A_1 and 192 to 208 from vendor A_2. The difference in range suggests that vendor A_2 may be preferred intuitively.

Confidence Intervals

The calculations shown in the ANOVA table are only estimates of the population parameters. These statistics are dependent on the size of the sample being investigated. As more castings

Table 6-7. ANOVA table for cylinder block castings from two sources—Example 6-2

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
Factor (A)	1	500.00	500.00	2.81	321.89	8.68
Error (e)	18	3206.00	178.00	1.00	3384.11	91.30
Total	19	3706.00				100.00

are sampled, the precision of the estimate would be improved. For large samples, the estimates approach the true value of the parameter. In statistics, it is therefore customary to represent the values of a statistical parameter as a range within which it is likely to fall, for a given level of confidence. This range is termed the confidence interval (C.I.). If the estimate of the mean value of a set of observations is denoted by $E(m)$, then the C.I. for the mean is given by:

$$\text{C.I.}(m) = E(m) \pm \sqrt{\frac{F_\alpha(f_1, f_2) \times V_e}{n_e}} \tag{6-15}$$

where

$F_\alpha(f_1, f_2)$ = variance ratio for DOF f_1 and f_2 at the level of significance α. The confidence level is $(1 - \alpha)$

f_1 = DOF of mean (which always equals 1)

f_2 = DOF of error term

V_e = variance of error term

n_e = number of equivalent replications, given by:

$$n_e = \frac{(\text{Number of trials})}{\begin{bmatrix}\text{DOF of mean (always 1)} + \\ \text{DOF of all factors used in the estimate}\end{bmatrix}}$$

To determine the C.I. for the estimated value of the mean for the above data, we proceed as follows:

$$E(m) = \begin{bmatrix} 240 + 190 + \ldots + 215 + 215 \\ +197 + 202 + \ldots + 195 + 201 \end{bmatrix} \Big/ 20$$

$$= 205$$

The number of experiments is 20, and there are two factors, m and A, involved in the estimates. Therefore,

$$n_e = \frac{20}{f_A + f_m} = \frac{20}{1+1} = 10$$

Because $F_{.1}\,(1,18) = 3.007$

and $V_e = 178.11$

$$m = 205 \pm \sqrt{\frac{3.007 \times 178.11}{10}}$$
$$= 205 \pm 7.32$$
$$= (197.68,\ 212.32)$$

Therefore, it can be stated that there is a 90% probability that the true value of the estimated mean will lie between 197.68 and 212.32.

The confidence interval can similarly be calculated for other statistics.

TWO-WAY ANOVA

The one-way ANOVA discussed above included one factor with two levels. This section extends ANOVA to experimental data of two or more factors with two or more levels. The following examples illustrate the procedure.

Example 6-3

The wear characteristics of two brands of tires (factor B), "Wearwell" and "Superwear," are to be compared. Several factors such as load, speed, and air temperature have significant effect on the useful life of tires. The problem will be limited to only one among these factors, that is, temperature (factor A). Let T_w and T_s represent winter (low) and summer (high) temperatures, respectively. Tire life (response characteristic) is measured in hours of operation at constant speed and load. The experiment design for this example is given in Table 6-8. This is called a two-factor two-level experiment. It has four possible trial runs, and the results of each run can be interpreted as follows:

With A at A_1 and B at B_1, the life is = 70 hr
With A at A_1 and B at B_2, the life is = 75 hr
With A at A_2 and B at B_1, the life is = 65 hr
With A at A_2 and B at B_2, the life is = 60 hr

The analysis of the data follows exactly the same procedures presented in the previous example. In this case, the total degrees

Table 6-8. Tire wear experiment—Example 6-3

		TEMPERATURE A		
	TIRE TYPE	$A_1 = T_w$	$A_2 = T_s$	SUM
B	"Wearwell"	70	65	135
	B_1	Y_1	Y_3	
	"Superwear"	75	60	135
	B_2	Y_2	Y_4	
	Sum =	145	125	270 = Grand total

of freedom, $f_T = n - 1 = 3$. The degrees of freedom and the ANOVA quantities in this case become:

$$
\begin{aligned}
n &= 2^2 = 4 \\
f_T &= n - 1 = 4 - 1 = 3 \\
f_A &= \text{number of levels} - 1 = 2 - 1 = 1 \\
f_B &= \text{types of tires} - 1 = 2 - 1 = 1 \\
f_{A \times B} &= 1 \times 1 = 1 \\
f_e &= f_T - f_A - f_B - f_{A \times B} \\
\text{C.F.} &= \text{correction factor} = T^2/n = 270^2/4 = 18225.0 \\
Y_0 &= \text{target value} = 0 \\
S_T &= \text{sum of squares of all results} - \text{C.F.} \\
&= \left(Y_1^2 + \cdots + Y_4^2\right) - \text{C.F.} \\
&= 70^2 + 65^2 + 75^2 + 60^2 - 18225.0 \\
&= 18350.0 - 18225.0 = 125.0
\end{aligned}
$$

The total contribution of each factor is calculated as follows:

$$
\begin{aligned}
A_1 &= 70 + 75 = 145 \qquad & A_2 &= 65 + 60 = 125 \\
B_1 &= 70 + 65 = 135 \qquad & B_2 &= 75 + 60 = 135
\end{aligned}
$$

and

$$S_A = A_1^2/N_{A_1} + A_2^2/N_{A_2} - \text{C.F.}$$

$$S_B = B_1^2/N_{B_1} + B_2^2/N_{B_2} - \text{C.F.}$$

$$S_{AB} = \sum_{i=1}^{2}\sum_{j=1}^{2} \left(A_i B_j\right)^2 / r_{ij} - \text{C.F.}$$

$$S_{A \times B} = S_{AB} - S_A - S_B$$
$$S_T = S_e + S_A + S_B + S_{A \times B}$$

where N_{A_1}, N_{A_2}, and so on, refer to the number of trial runs included in the sums A_1, A_2, and so on. A_iB_j is the total experimental response for factor A at level i and factor B at level j whereas r_{ij} is the number of replications (observations) for cell ij. The term $S_{A\times B}$ represents the interaction sum of squares.

For the above example,

$$S_A = 145^2/2 + 125^2/2 - 18225.0 = 18325.0 - 18225.0 = 100.0$$
$$S_B = 135^2/2 + 135^2/2 + 18225.0 = 18225.0 - 18225.0 = 0.0$$

Because $r = 1$ (one observation per cell)

$$S_{AB} = 70^2/1 + 75^2/1 + 65^2/1 + 60^2/1 - 18225.0 = 125.0$$

and

$$S_{A \times B} = 125.0 - 100.0 - 0 = 25.0$$

Therefore, using Eq. (6-11), the error variation becomes,

$$S_e = S_T - (S_A + S_B + S_{A\times B})$$
$$= 125 - 100 - 0 - 25 = 0$$

Variance calculations:

$$V_A = S_A/f_A \quad = 100/1 = 100$$
$$V_B = S_B/f_B \quad = 0/1 \quad = 0$$
$$V_{A\times B} = S_{A\times B}/f_{A\times B} = 25/1 \quad = 25$$
$$V_e = S_e/f_e \quad = 0/0 \quad \text{indeterminate, hence not useful}$$

Variance Ratio

Once all of the variances are computed, the results can be arranged in tabular form, as appears in Table 6-9.

Observe that the DOF and S_e of the error terms are zero, hence F, the ratio of the variances, cannot be computed. Thus this experimental design is not effective for studying the interaction of factors A and B. Additional degrees of freedom are necessary for

Table 6-9. ANOVA table for tire wear characteristics—Example 6-3

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
A	1	100.00	100.00			
B	1	0.00	0.00			
$A \times B$	1	25.00	25.00			
Error (e)	0	0.00				
Total	3	125.00				

a complete analysis of the interactions and main effects. This can be accomplished by repeating the observations for each setup so that there will be an error term that will have non-zero degrees of freedom and variance terms.

EXPERIMENTS WITH REPLICATIONS

Example 6-4

Example 6-3 is extended to two observations per cell, as shown in Table 6-10. In Table 6-11, the data in each cell are replaced by a single value obtained by adding the two data points. The total of the degrees of freedom, f_T, is increased:

because $\quad n = r \times 2^2 = 2 \times 4 = 8$

and $\quad f_T = n - 1 = 8 - 1 = 7$

The degrees of freedom for other factors are as follows:

$$f_A = \text{number of levels} - 1 = 2 - 1 = 1$$
$$f_B = \text{types of tires} - 1 = 2 - 1 = 1$$
$$f_{A\times B} = 1 \times 1 = 1$$
$$f_e = f_T - f_A - f_B - f_{A\times B} = 7 - 1 - 1 - 1 = 4$$

and

$$\text{C.F.} = \text{correction factor} = T^2/n = 542^2/8 = 36720.5$$

Table 6-10. Tire wear experiments with repetitions—Example 6-4

	TIRE TYPE	TEMPERATURE A: $A_1 = T_w$	$A_2 = T_s$
B	"Wearwell" B_1	70, 72 Y_1	65, 62 Y_3
	"Superwear" B_2	75, 77 Y_2	60, 61 Y_4

Table 6-11. Tire wear experiments with repetitions—Example 6-4

	TIRE TYPE	TEMPERATURE A: $A_1 = T_w$	$A_2 = T_s$	SUM
B	"Wearwell" B_1	142 Y_1	127 Y_3	269
	"Superwear" B_2	152 Y_2	121 Y_4	273
	Sum =	294	248	542 = Grand total

Assuming

Y_0 = target value = 0

S_T = sum of squares of all eight data points – C.F.

$$= \left(Y_1^2 + \cdots + Y_8^2\right) - \text{C.F.}$$
$$= 70^2 + 72^2 + 75^2 + 77^2 + 65^2 + 62^2 + 60^2 + 61^2 - 36720.5$$
$$= 37028.0 - 36720.5$$
$$= 307.5$$

The contribution of each factor is shown below:

$A_1 = 142 + 152 = 294 \qquad A_2 = 127 + 121 = 248$

$B_1 = 142 + 127 = 269 \qquad B_2 = 152 + 121 = 273$

and $S_A = 294^2/4 + 248^2/4 - 36720.5 = 264.5$

$$S_B = 269^2/4 + 273^2/4 - 36720.5 = 2.0$$

$$S_{A\times B} = (A_1B_1)^2/2 + (A_1B_2)^2/2 + (A_2B_1)^2/2 + (A_2B_2)^2/2 - \text{C.F.}$$

$$= \left[(142)^2 + (152)^2 + (127)^2 + (121)^2\right]/2 - 36720.5$$

$$= 37019 - 36720.5 = 298.5$$

Also $S_{A\times B} = 298.5 - 264.5 - 2 = 32$

$$S_e = S_T - (S_A + S_B + S_{A\times B})$$

$$= 307.5 - 264.5 - 2 - 32 = 9.0$$

Variance calculations:

$$V_A = S_A/f_A = 264.5/1 = 264.5$$

$$V_B = S_B/f_B = 2/1 = 2$$

$$V_{A\times B} = S_{A\times B}/f_{A\times B} = 32/1 = 32$$

$$V_e = S_e/f_e = 9/4 = 2.25$$

Also $F_A = V_A/V_e = 264.5/2.25 = 117.6$

$$F_B = V_B/V_e = 2/2.25 = 0.89$$

$$F_{A\times B} = V_{A\times B}/V_e = 32/2.25 = 14.22$$

Pure sum of squares:

$$S'_A = S_A - f_A \times V_e = 264.5 - 1 \times 2.25 = 262.25$$

$$S'_B = S_B - f_B \times V_e = 2 - 1 \times 2.25 = -0.25$$

$$S'_{A\times B} = S_{A\times B} - f_{A\times B} \times V_e = 32 - 1 \times 2.25 = 29.75$$

$$S'_e = S_e + (f_A + f_B + f_{A\times B}) \times V_e = 9 + 3 \times 2.25 = 15.75$$

and the percentage contributions will be:

$$P_A = S'_A \times 100/S_T = 262.25 \times 100/307.5 = 85.28$$

$$P_B = S'_B \times 100/S_T = -0.25 \times 100/307.5 = -0.08$$

$$P_{A\times B} = S'_{A\times B} \times 100/S_T = 29.75 \times 100/307.5 = 9.68$$

$$P_e = S'_e \times 100/S_T = 15.75 \times 100/307.5 = 5.12$$

Table 6-12. ANOVA table for tire wear with repetitions—Example 6-4

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
A	1	264.50	264.50	117.50	262.25	85.28
B	1	2.00	2.00	0.89	–0.25	–0.08
$A \times B$	1	32.00	32.00	14.22	29.75	9.68
Error (e)	4	9.00	2.25	1.00	15.75	5.12
Total	7	307.50				100.00

Because the number of the degrees of freedom for the numerator is 1 (see Table 6-12) and that for the denominator is 4, from the F-tables at .05 level of significance (95% confidence) we obtain $F_{.05}$ (1,4) = 7.7086. The computed values of variance ratios F for factor A and interaction $A \times B$ are greater than the limiting values obtained from the table. Therefore, there is a significant difference in the wear life of the tires under summer and winter conditions. The interaction term, $F_{A \times B}$, indicates that the influence of temperature on the two brands of tires is also significant. However, F_B is less than the F-table factor. Thus, there is no difference between tire brands within the confidence level.

Procedures for Pooling

When the contribution of a factor is small, as for factor B in the above example, the sum of squares for that factor is combined with the error, S_e. This process of disregarding the contribution of a selected factor and subsequently adjusting the contributions of the other factor is known as *pooling*. Pooling is usually accomplished by starting with the smallest sum of squares and continuing with the ones having successively larger effects. Pooling is recommended when a factor is determined to be insignificant by performing a test of significance against the error term at a desired confidence level. A general guideline for when to pool is obtained by comparing the error DOF with the total factor DOF. Taguchi recommends pooling factors until the error DOF is approximately half the total DOF of the experiment ([9], pp. 293-295). Approaching the matter technically, one could test for significance and pool all factor influences below the 90% confidence level. The procedure for significance testing will be discussed later in this chapter. For now, we will arbitrarily select small factor effects and pool. Consider the pooling effects of factor B. If the variance for this factor is pooled with the error term, the new error variance is computed as:

$$\begin{aligned} V_e &= (S_B + S_e)/(f_B + f_e) \\ &= (2.0 + 9.0)/(1 + 4) \\ &= 2.2 \end{aligned}$$

With a pooled V_e, all S' values must be modified to reflect pooling:

$$S'_A = S_A - (V_e \times f_A) = 264.50 - 2.20 = 262.30$$

$$S'_{A\times B} = S_{A\times B} - (V_e \times f_{A\times B}) = 32.0 - 2.2 \times 1 = 29.8$$

$$S'_e = S_e + V_e\,(f_A + f_{A\times B}) = 11.0 + 2.2 \times 2 = 15.4$$

The results of this procedure are summarized in Table 6-13, which makes it apparent that pooling in this particular case does not appreciably change the results. But, in certain cases, the process may significantly affect the results. No matter the effect on the results, insignificant factors should always be pooled.

Table 6-13. ANOVA table for tire wear with repetitions and pooling—Example 6-4

SOURCE OF VARIATION	f	SUM OF SQUARES	VARIANCE (MEAN SQUARE), V	VARIANCE RATIO, F	PURE SUM OF SQUARES, S'	PERCENT CONTRIBUTION, P
A	1	264.50	264.50	120.20	262.30	85.30
B			Pooled			
$A \times B$	1	32.00	32.00	14.50	29.80	9.69
Error (e)	5	11.00	2.20	1.00	15.40	5.01
Total	7	307.50				100.00

It is quite evident from these considerations that the ANOVA procedure is cumbersome and extremely time consuming. The computations necessary increase tremendously as the size of the matrix increases. The design of the experiment and the subsequent analysis of the test results can be simplified using available software specifically made for analysis of Taguchi experimental designs. Most of the computations shown in this book have been carried out using Qualitek-4 software [7]. To further clarify the step-by-step procedure involved in analysis of variance, the following numerical example is presented.

STANDARD ANALYSIS WITH SINGLE AND MULTIPLE RUNS

Example 6-5

A Taguchi experiment was designed (Table 6-14) to investigate five two-level factors (A, B, C, D, and E) and two interactions ($A \times C$ and $B \times C$) of a certain manufacturing operation. The L_8 orthogonal array was used to design the experiment, and the results were examined of one sample that was tested under each experimental configuration. The results are shown in Table 6-15.

Analysis Using Single Run

Level Totals and Their Averages

The factor averages at each factor level are obtained by adding the results of all trial conditions at the level considered and then dividing by the number of data points added.

$$\bar{A}_1 = (y_1 + y_2 + y_3 + y_4)/4 = (42 + 50 + 36 + 45)/4$$
$$= 173/4 = 43.25$$

$$\bar{A}_2 = (y_5 + y_6 + y_7 + y_8)/4 = (35 + 55 + 30 + 54)/4$$
$$= 174/4 = 43.50$$

$$\bar{C}_1 = (y_1 + y_2 + y_5 + y_6)/4 = (42 + 50 + 35 + 55)/4$$
$$= 182/4 = 45.50$$

$$\bar{C}_2 = (y_3 + y_4 + y_7 + y_8)/4 = (36 + 45 + 30 + 54)/4$$
$$= 165/4 = 41.25$$

Table 6-14. Factors and their levels—Example 6-5

COLUMN	FACTOR NAMES	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Factor *A*	A_1	A_2		
2	Factor *C*	C_1	C_2		
3	Interaction $A \times C$	N/A			
4	Factor *B*	B_1	B_2		
5	Factor *D*	D_1	D_2		
6	Interaction $B \times C$	N/A			
7	Factor *E*	E_1	E_2		

Objective: Determine best design parameters.
Characteristic: Smaller is better.

Table 6-15. Layout and results—Example 6-5

FACTORS	*A*	*C*	$A \times C$	*B*	*D*	$B \times C$	*E*	RESULTS
COLUMN	1	2	3	4	5	6	7	(*y*)
TRIAL								
Trial 1	1	1	1	1	1	1	1	42.00
Trial 2	1	1	1	2	2	2	2	50.00
Trial 3	1	2	2	1	1	2	2	36.00
Trial 4	1	2	2	2	2	1	1	45.00
Trial 5	2	1	2	1	2	1	2	35.00
Trial 6	2	1	2	2	1	2	1	55.00
Trial 7	2	2	1	1	2	2	1	30.00
Trial 8	2	2	1	2	1	1	2	54.00
								Total = 347.00

Similarly

$$B_1 = 143 \qquad \bar{B}_1 = 35.75$$
$$B_2 = 204 \qquad \bar{B}_2 = 51.00$$
$$D_1 = 187 \qquad \bar{D}_1 = 46.75$$
$$D_2 = 160 \qquad \bar{D}_2 = 40.00$$
$$E_1 = 172 \qquad \bar{E}_1 = 43.00$$
$$E_2 = 175 \qquad \bar{E}_2 = 43.75$$

and $\overline{(A \times C)}_1 = (y_1 + y_2 + y_7 + y_8)/4 = 176/4 = 44.00$

$\overline{(A \times C)}_2 = (y_3 + y_4 + y_5 + y_6)/4 = 171/4 = 42.75$

$\overline{(B \times C)}_1 = (y_1 + y_4 + y_5 + y_8)/4 = 176/4 = 44.00$

$\overline{(B \times C)}_2 = (y_2 + y_3 + y_6 + y_7)/4 = 171/4 = 42.75$

The results are shown in Table 6-16, and the main effects are plotted in Figure 6-1.

Ignoring interaction effects and assuming the "smaller is better" characteristic is desired, the optimum condition becomes:

$A_1 \quad C_2 \quad B_1 \quad D_2 \quad E_1$

Computation of Interaction

Interaction effects are always mixed with the main effects of the factors assigned to the column designated for interaction. The relative significance of the interaction effects is obtained by ANOVA, just as are the relative significance of factor effects. To determine whether two factors, A and C, interact, the following calculations are performed.

Level totals and their averages for A and C:

$\overline{A_1C_1} = (y_1 + y_2)/2 = (42 + 50)/2 = 92/2 = 46.0$

$\overline{A_1C_2} = (y_3 + y_4)/2 = (36 + 45)/2 = 81/2 = 40.5$

$\overline{A_2C_1} = (y_5 + y_6)/2 = (35 + 55)/2 = 90/2 = 45.0$

$\overline{A_2C_2} = (y_7 + y_8)/2 = (30 + 54)/2 = 84/2 = 42.0$

Level totals and their averages for B and C:

$\overline{B_1C_1} = (y_1 + y_5)/2 = (42 + 35)/2 = 77/2 = 38.5$

$\overline{B_1C_2} = (y_3 + y_7)/2 = (36 + 30)/2 = 66/2 = 33.0$

$\overline{B_2C_1} = (y_2 + y_6)/2 = (50 + 55)/2 = 105/2 = 52.5$

$\overline{B_2C_2} = (y_4 + y_8)/2 = (45 + 54)/2 = 99/2 = 49.5$

Table 6-16. Average effects—Example 6-5

COLUMN	FACTOR NAMES	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Factor *A*	43.25	43.50	0.25		
2	Factor C	45.50	41.25	–4.25		
3	Interaction $A \times C$	44.00	42.75	–1.25		
4	Factor *B*	35.75	51.00	15.25		
5	Factor *D*	46.75	40.00	–6.75		
6	Interaction $B \times C$	44.00	42.75	–1.25		
7	Factor *E*	43.00	43.75	0.75		

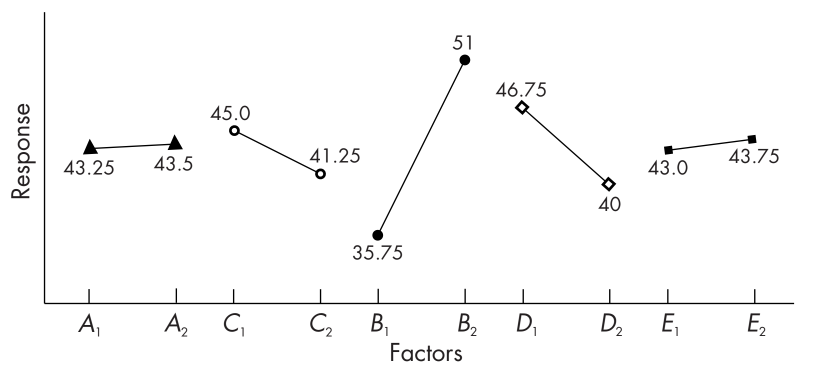


Figure 6-1. Main effects—Example 6-5

These items represent the combined effects of the factors on the results. For example, A_1C_1 represents the effect of factor A at level 1 and C at level 1 together.

These results are plotted in Figure 6-2 by selecting factor C, which is common to both pairs of interactions, arbitrarily to represent the x-axis. The angle between the two lines in this interaction plot indicates the strength of presence of interaction (need not intersect). Note that lines B_1 and B_2 appear almost parallel; hence, B and C interact slightly. Note also that A_1 and A_2 intersect; thus, A and C interact.

The minor interaction of $B \times C$ is ignored, but interaction $A \times C$ is included in the optimum condition. For the "smaller is

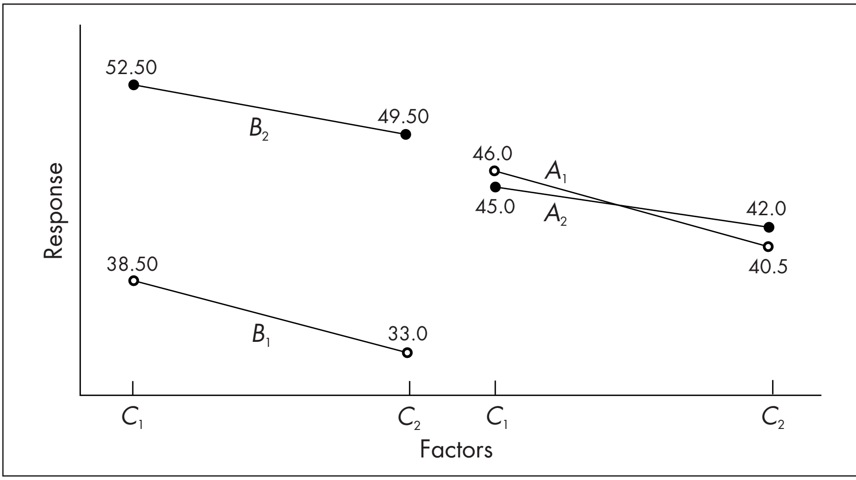


Figure 6-2. Interaction A × C and B × C—Example 6-5

better" situation, A_1C_2 (40.5) produces the lowest average value. Therefore, A_1C_2 is must be included in the optimum condition. It so happens that A_1C_2 was already included in the optimum condition, $A_1\ C_2\ B_1\ D_2\ E_1$, based on the main effects alone.

Analysis of Variance (ANOVA)

ANOVA establishes the relative significance of the individual factors and the interaction effects. The steps are as follows:

Step 1. Total of all results:

$$T = 42 + 50 + 36 + 45 + 35 + 55 + 30 + 54 = 347$$

Step 2. Correction Factor:

$$\text{C.F.} = T^2/n = 347^2/8 = 15051.125$$

Note: n = total number of experiments, 8.

Step 3. Total sum of squares:

$$S_T = \sum_{i=1}^{8} y_i^2 - \text{C.F.}$$

$$= \left(42^2 + 50^2 + 36^2 + \ldots + 54^2\right) - 15051.125$$

$$= 599.88$$

Step 4. Factor sum of squares:

$$S_A = A_1^2/N_{A_1} + A_2^2/N_{A_2} - \text{C.F.}$$
$$= 173^2/4 + 174^2/4 - 15051.125 = 0.125$$
$$S_B = B_1^2/N_{B_1} + B_2^2/N_{B_2} - \text{C.F.}$$
$$= 143^2/4 + 204^2/4 - 15051.125 = 465.125$$
$$S_C = C_1^2/N_{C_1} + C_2^2/N_{C_2} - \text{C.F.}$$
$$= 182^2/4 + 165^2/4 - 15051.125 = 36.125$$
$$S_D = D_1^2/N_{D_1} + D_2^2/N_{D_2} - \text{C.F.}$$
$$= 187^2/4 + 160^2/4 - 15051.125 = 91.125$$
$$S_E = E_1^2/N_{E_1} + E_2^2/N_{E_2} - \text{C.F.}$$
$$= 172^2/4 + 175^2/4 - 15051.125 = 1.125$$
$$S_{A\times C} = (A\times C)_1^2/N_{(A\times C)_1} + (A\times C)_2^2/N_{(A\times C)_2} - \text{C.F.}$$
$$= 176^2/4 + 171^2/4 - 15051.125 = 3.125$$
$$S_{B\times C} = (B\times C)_1^2/N_{(B\times C)_1} + (B\times C)_2^2/N_{(B\times C)_2} - \text{C.F.}$$
$$= 176^2/4 + 171^2/4 - 15051.125 = 3.125$$

Alternative Formula for Two-Level Factors

In the case of two-level factors, the sums of squares can be computed using the following formulas:

$$S_A = \frac{(A_1 - A_2)^2}{(N_{A_1} + N_{A_2})} = \frac{(173-174)^2}{(4+4)} = 0.125$$

$$S_B = \frac{(B_1 - B_2)^2}{(N_{B_1} + N_{B_2})} = \frac{(143-204)^2}{(4+4)} = 465.125$$

$$S_C = \frac{(C_1 - C_2)^2}{(N_{C_1} + N_{C_2})} = \frac{(182-165)^2}{(4+4)} = 36.125$$

$$S_D = \frac{(D_1 - D_2)^2}{(N_{D_1} + N_{D_2})} = \frac{(187-160)^2}{(4+4)} = 91.125$$

$$S_E = \frac{(E_1 - E_2)^2}{(N_{E_1} + N_{E_2})} = \frac{(172-175)^2}{(4+4)} = 1.125$$

$$S_{A\times C} = \frac{((A\times C)_1 - (A\times C)_2)^2}{(N_{(A\times C)_1} + N_{(A\times C)_2})} = \frac{(176-171)^2}{(4+4)} = 3.125$$

$$S_{B\times C} = \frac{((B\times C)_1 - (B\times C)_2)^2}{(N_{(B\times C)_1} + N_{(B\times C)_2})} = \frac{(176-171)^2}{(4+4)} = 3.125$$

$$\begin{aligned} S_e &= S_T - (S_A + S_B + S_C + S_D + S_E + S_{A\times C} + S_{B\times C}) \\ &= 599.88 - \begin{pmatrix} 0.125 + 465.125 + 36.125 + 91.125 \\ +91.125 + 3.125 + 3.125 \end{pmatrix} \\ &= 599.88 - 599.88 = 0 \end{aligned}$$

where

N_{A_1} = total number of experiments where factor A_1 is present
N_{B_1} = total number of experiments where factor B_1 is present
A_1 = sum of results (Y_i) where factor A_1 is present
B_1 = sum of results (Y_i) where factor B_1 is present

Step 5. Total and factor degrees of freedom (DOF):

DOF total = number of test runs minus 1

or $f_T = n - 1 = 8 - 1 = 7$

DOF of each factor is 1 less than the number of levels:

f_A = (number of levels of factor A) – 1
= 2 – 1 = 1

f_B = (number of levels of factor B) – 1
= 2 – 1 = 1

f_C = (number of levels of factor C) – 1
= 2 – 1 = 1

f_D = (number of levels of factor D) – 1
$= 2 - 1 = 1$
f_E = (number of levels of factor E) – 1
$= 2 - 1 = 1$

Also,

$$f_{(A \times C)} = f_A \times f_C$$
$$= 1 \times 1 = 1$$
$$f_{(B \times C)} = f_B \times f_C$$
$$= 1 \times 1 = 1$$

DOF of the error term in this example:

$$f_e = f_T - \left(f_A + f_B + f_C + f_D + f_E + f_{A\times C} + f_{B\times C}\right)$$
$$= 7 - 7 = 0$$

With the error degrees of freedom equal to zero, $f_e = 0$, information regarding the error sum of squares cannot be determined. In addition, F ratios for factors cannot be calculated because the calculations involve f_e. To complete the calculations, smaller factorial effects are added together (pooled) to form a new non-zero estimate of the error term. This is discussed in the following section.

Step 6. Mean square (variance):

$$V_A = S_A / f_A = 0.125/1 = 0.125$$
$$V_B = S_B / f_B = 465.125/1 = 465.125$$
$$V_C = S_C / f_C = 36.125/1 = 36.125$$
$$V_D = S_D / f_D = 91.125/1 = 91.125$$
$$V_E = S_E / f_E = 1.125/1 = 1.125$$
$$V_e = S_e / f_e = 0/0 = \text{indeterminate}$$

As the variance of error term (V_e) is zero, the variance ratio and pure sum of squares (S') cannot be calculated. In this case, the percentage contributions are first calculated using sums of squares. Then, if there are insignificant factors, pool them and recalculate percentages using the pure sums of squares.

Step 7. Percentage contribution:

$$P_A = S_A/S_T = 0.125/599.88 = 0.02$$
$$P_B = S_B/S_T = 465.125/599.88 = 77.54$$
$$P_C = S_C/S_T = 36.125/599.88 = 6.02$$
$$P_D = S_D/S_T = 91.125/599.88 = 15.20$$
$$P_E = S_E/S_T = 1.125/599.88 = 0.19$$
$$P_{A\times C} = S_{A\times C}/S_T = 3.125/599.88 = 0.52$$
$$P_{B\times C} = S_{B\times C}/S_T = 3.125/599.88 = 0.52$$

and P_e cannot be calculated because $V_e = 0$.

The results of the analysis of variance are summarized in Table 6-17.

Pooling

Note that in Step 7 the effects of factors A and E and interactions $B\times C$ and $A\times C$ are small, totaling slightly more than 1% (1.2%). These factors are pooled to obtain new, non-zero estimates of S_e and f_e.

Sum of squares of error term:

Let: $S_e = S_A + S_E + S_{A\times C} + S_{B\times C}$

then $S_e = S_T - (S_B + S_C + S_D) = 599.9 - (592.4) = 7.5$

Table 6-17. ANOVA table—Example 6-5

COLUMN	FACTOR NAMES	*f*	*S*	*V*	*F*	*P*
1	Factor *A*	1	0.125	0.125		0.02
2	Factor C	1	36.125	36.125		6.02
3	Interaction *A* × C	1	3.125	3.125		0.52
4	Factor *B*	1	465.125	465.125		77.54
5	Factor *D*	1	91.125	92.125		15.20
6	Interaction *B* × C	1	3.125	3.125		0.52
7	Factor *E*	1	1.125	1.125		0.19
All other/error		0	0	0		
Total		7	599.880			100.00%

Degree of freedom of error term:

$$f_e = f_T - (f_B + f_C + f_D)$$
$$= 7 - 3 = 4$$

Variance of error term:

$$V_e = S_e / f_e = 7.5/4 = 1.875$$

Factor F ratios, for significant factors:

$$F_C = V_C / V_e = 36.125/1.875 = 19.267$$
$$F_B = V_B / V_e = 465.125/1.875 = 248.067$$
$$F_D = V_D / V_e = 91.125/1.875 = 48.600$$

Pure sum of squares, S', for significant factors:

$$S'_C = S_C - (V_e \times f_C)$$
$$= 36.125 - (1.875 \times 1) = 34.25$$
$$S'_B = S_B - (V_e \times f_B)$$
$$= 465.125 - (1.875 \times 1) = 463.25$$
$$S'_D = S_D - (V_e \times f_D)$$
$$= 91.125 - (1.875 \times 1) = 89.25$$

Note that in the ANOVA in Table 6-18, the pure sum of squares, S', is not shown.

Percentage contribution:

$$P_C = S'_C / S_T = 34.25/599.88 = 5.71\%$$
$$P_B = S'_B / S_T = 463.25/599.88 = 77.22\%$$
$$P_D = S'_D / S_T = 89.25/599.88 = 14.88\%$$
$$P_e = 100\% - (P_C + P_B + P_D) = 100\% - (5.71 + 77.22 + 14.88)$$
$$= 100\% - 97.81 = 2.19\%$$

The ANOVA terms that are modified after pooling are shown in Table 6-18.

Taguchi's guideline for pooling ([9], pp. 293-295) requires starting with the smallest main effect and successively including

Table 6-18. Pooled ANOVA—Example 6-5

COLUMN	FACTOR NAMES	*f*	*S*	*V*	*F*	*P*
1	Factor *A*	(1)	(0.13)	Pooled		
2	Factor C	1	36.125	36.125	19.267	5.71
3	Interaction $A \times C$	(1)	(3.125)	Pooled		
4	Factor *B*	1	465.125	265.125	248.067	77.22
5	Factor *D*	1	91.125	92.125	48.600	14.88
6	Interaction $B \times C$	(1)	(3.125)	Pooled		
7	Factor *E*	(1)	(1.125)	Pooled		
All other/error		4	7.500	1.88		2.19
Total		7	599.880			100.00%

larger effects until the total pooled DOF equals approximately half of the total DOF. The larger DOF for the error term, as a result of pooling, increases the confidence level of the significant factors.

Note that as small factor effects are pooled, the percentage contributions and the confidence level of the remaining factors decrease (P_C = 5.71 versus P_C = 6.02). By pooling, the error term is increased and, in comparison, the other factors appear less influential. The greater the number of factors pooled, the worse the unpooled factor effects look. Then we must consider why column effects are pooled.

Error variance represents the degree of inter-experiment error when the DOF of the error term is sufficiently large. When the error DOF is small or zero, which is the case when all columns of the OA are occupied and trials are not repeated, small column effects are successively pooled to form a larger error term (this is known as a pooling-up strategy). The factors and interactions that are now significant, in comparison with the larger magnitude of the error term, are now influential. Taguchi prefers this strategy as it tends to avoid the mistake (alpha mistake) of ignoring helpful factors.

A large error DOF naturally results when trial conditions are repeated and standard analysis is performed. When the error DOF is large, pooling may not be necessary. Therefore, one could repeat the experiment and avoid pooling, but to repeat all trial conditions just for information on the error term may not be practical.

A sure way to determine if a factor or interaction effect should be pooled is to perform a test of significance (1 – confidence level). But what level of confidence do you work with? No clear guidelines are established. Generally, factors are pooled if they do not pass the test of significance at the confidence level assumed for the experiment. A factor is considered significant if its experimental F-ratio exceeds the standard table value at a confidence level. A common practice is to subjectively assume a confidence level between 85% and 99%, with 90% or 95% being a popular selection. Consider factor C in *Example 6-5*, which has 5.7% influence (19.267 F-ratio). When tested for significance, this factor shows more than 99% confidence level and thus should not be pooled.

From the ANOVA table

$$F_C = 19.267$$

From the F-table, find the F value at

$$n_1 = \text{DOF of factor } C = 1$$
$$n_2 = \text{DOF of error term} = 4$$

at a confidence level (say, the 99% confidence level).

$$F = 21.198 \text{ (from Table C-4)}$$

As F_C from the experiment (19.267) is smaller than the F-table value (21.198), factor C should be pooled.

SUMMARY RESULTS

Description of the factor	= Factor C
Column the factor is assigned to	= 2
Variance ratio for this factor	= 19.267
DOF of the factor	= 1
DOF of error term, f_e	= 4
Confidence level %	= 99

Based on the level of confidence desired (99%), the following is recommended:

"Pool this factor"

The revised values are calculated as shown below:

$$S_e = S_T - (S_B + S_D) = 599.9 - (556.25) = 43.625$$

$$f_e = f_T - (f_B + f_D) = 7 - 2 = 5$$

$$V_e = S_e / f_e = 43.625/5 = 8.725$$

$$F_B = V_B / V_e = 465.125/8.725 = 53.309$$

$$F_D = V_D / V_e = 91.125/8.725 = 10.444$$

$$S'_B = S_B - (V_e \times f_B) = 465.125 - (8.725 \times 1) = 456.40$$

$$S'_D = S_D - (V_e \times f_D) = 91.125 - (8.725 \times 1) = 82.40$$

$$P_B = S'_B / S_T = (456.40 \times 100)/599.88 = 76.08$$

$$P_D = S'_D / S_T = (82.40 \times 100)/599.88 = 13.74$$

$$P_e = 100\% - (P_B + P_D) = 100\% - (76.08 + 13.74) = 10.18$$

Confidence Interval of Factor Effect

The confidence interval of estimates of the main effect is calculated using the following expression:

$$\text{C.I.} = \pm\sqrt{\left(F(1, n_2) \times V_e / N_e\right)}$$

where

$F\ (1,n_2)$ = F value from the F-table at a required confidence level and at DOF 1 and error DOF n_2

V_e = variance of error term (from ANOVA)

N_e = effective number of replications

$$= \frac{\text{Total number of results (or number of S/N ratios)}}{\text{DOF of mean (=1, always) + DOF of all factors included in the estimate of the mean}}$$

Thus for factor C at level C_1, the C.I. is calculated by first determining the F factor:

n_2 = 4

N_e = 8/(1+1) = 4

$F\ (1,4)$ = 7.7086 at 95% confidence level

then C.I. = ±1.9034 at 95% confidence level
because C_1 = 45.50 (Table 6-16)
expected value of C_1 = 45.50 ± 1.9034

SUMMARY RESULTS

Based on:

F value from the table (at a confidence level) = 7.7086
Error variance, V_e = 1.88
Number of effective replications = 4

The confidence interval C.I. is calculated as follows:

$$\text{C.I.} = \pm\sqrt{\left(F\left(1, n_2\right) \times V_e / N_e\right)}$$

C.I. represents the boundaries of the expected performance in the optimum condition at a confidence level used for the *F* value from the standard table.

Confidence interval (C.I.) = ±1.9034

Estimated Result at Optimum Condition

The performance at the optimum condition is estimated only from the significant factors. This practice keeps the predicted performance conservative. Therefore, the pooled factors are not included in the estimate.

Grand average of performance: $\bar{T} = 347/8 = 43.375$

As factors *B, C,* and *D* are considered significant, the performance at the optimum condition will be estimated using only these three factors.

$$= \bar{T} + \left(\bar{B}_1 - \bar{T}\right) + \left(\bar{D}_2 - \bar{T}\right) + \left(\bar{C}_2 - \bar{T}\right)$$

$$= 43.375 + \left(35.75 - 43.375\right) + \left(40 - 43.375\right) + \left(41.25 - 43.375\right)$$

$$= 30.25$$

Note that the optimum condition for the "smaller is better" quality characteristic is $B_1\,C_2\,D_2$. The average values at these conditions were previously calculated as summarized in Table 6-19.

Table 6-19. Estimate of performance at optimum condition—Example 6-5

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Factor *C*	C_2	2	–2.125
Factor *B*	B_1	1	–7.6250
Factor *D*	D_2	2	–3.3750
Contribution from all factors (total)			–13.125
Current grand average of performance			43.375
Expected result at optimum condition			30.250

Confidence Interval of the Result at the Optimum Condition

The expression for computing the confidence interval, for performance at the optimum condition, is calculated in the same way as are the factor effects.

$$\text{C.I.} = \pm\sqrt{\left(F(1,n_2) \times V_e / N_e\right)}$$

where

$F\,(1,n_2)$ = F value from the F-table at a required confidence level and at DOF 1 and error DOF n_2

V_e = variance of error term (from ANOVA)

N_e = effective number of replications

$$= \frac{\text{Total number of results (or number of S/N ratios)}}{\text{DOF of mean (=1, always) + DOF of all factors included in the estimate of the mean}}$$

Three factors, B_1, C_2, and D_2, are included in calculating the estimate of the performance at the optimum condition. Therefore, the effective number of replications, the F value, and the confidence intervals are calculated as shown below. A confidence level of 85% to 99% is the normal range of selection for common industrial experiments. A 90% confidence level is arbitrarily selected for the following calculations.

$n_2 = 4$
$N_e = 8/(1+3) = 2$
$F\ (1,4) = 4.5448$ at the 90% confidence level
$V_e = 1.88$
C.I. $= \pm 2.067$ at the 90% confidence level

Therefore, the result at the optimum condition is 22.0 ± 2.067 at the 90% confidence level.

SUMMARY RESULTS

Expression: $\text{C.I.} = \pm\sqrt{\left(F\left(1,n_2\right)\times V_e/N_e\right)}$

where

$F\ (n_1, n_2)$ = computed value of F with $n_1 = 1$, n_2 = error DOF, at a desired confidence level
V_e = error variance
N_e = effective number of replications

Based on $F = 4.5448$, $n_1 = 1$ and $n_2 = 4$, $V_e = 1.88$, and $N_e = 2$:

The confidence interval (C.I.) = ±2.067

which is the variation of the estimated result at the optimum, that is, the mean result (m) lies between (m + C.I.) and (m − C.I.) at 90% confidence level.

The confidence interval formula assumes a sufficiently large number of data points so that the sample approximates the population characteristics. With a "small" sample, its characteristics may deviate from the population. When the sample is small, that is, only a finite number of confirmation tests is planned, the C.I. of the expected result is expressed as:

$$\text{C.I.} = \pm\sqrt{\left[F\left(n_1,n_2\right)\times V_e\times\left(N_e+N_r\right)\right]/N_e\times N_r}$$

where

$F\ (n_1, n_2)$ = computed value of F at a desired confidence level with $n_1 = 1$, n_2 = error DOF
V_e = error variance

N_e = effective number of replications
N_r = number of repetitions

Based on:

$n_1 = 1 \qquad n_3 = 4 \qquad V_e = 1.88$
$N_e = 2 \qquad N_r = 3$

$F_{.1}$ (1,4) = 4.5448 at the 90% confidence level

then C.I. = ±2.668 at the 90% confidence level. This is a wider interval than previously calculated based on a large number of repetitions.

Analysis with Multiple Runs

Assume the trial runs of the experiment were each repeated three times and that the average result of each trial as shown Table 6-20 is the same as that for a single trial in Table 6-15. The analysis takes a slightly different form. Because the averages of these hypothetical results were kept the same, the main effects remain unchanged, as shown in Tables 6-21 and 6-16. However, the results of ANOVA (Table 6-22) are significantly different from the corresponding result of the single run (Table 6-18).

Computation for ANOVA:

$$\begin{aligned} \text{DOF} &= \text{total number of results} - 1 \\ &= \text{number of trials} \times \text{number of repetitions} - 1 \\ &= 8 \times 3 - 1 = 23 \end{aligned}$$

Sample calculations using factor B:

$$\begin{aligned} B_1 = &(38.0 + 42.0 + 46.0) + (38.0 + 36.0 + 34.0) \\ &+ (30.0 + 35.0 + 40.0) + (40.0 + 30.0 + 20.0) = 429 \end{aligned}$$

Note that the trial condition for calculating B_1 is (1, 3, 5, and 7).

$$\begin{aligned} B_2 = &(45.0 + 50.0 + 55.0) + (55.0 + 45.0 + 35.0) \\ &+ (65.0 + 55.0 + 45.0) + (58.0 + 54.0 + 50.0) = 612 \end{aligned}$$

Using the sum of squares formula for a two-level factor,

Table 6-20. Results with three repetitions—Example 6-5

REPETITION TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	38.00	42.00	46.00				42.00
2	45.00	50.00	55.00				50.00
3	38.00	36.00	34.00				36.00
4	55.00	45.00	35.00				45.00
5	30.00	35.00	40.00				35.00
6	65.00	55.00	45.00				55.00
7	40.00	30.00	20.00				30.00
8	58.00	54.00	50.00				54.00

Table 6-21. Main effects—Example 6-5

COLUMN	FACTOR NAMES	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Factor *A*	43.25	43.50	0.25		
2	Factor *C*	45.50	41.25	–4.25		
3	Interaction *A* × *C*	44.00	42.75	–1.25		
4	Factor *B*	35.75	51.00	15.25		
5	Factor *D*	46.75	40.00	–6.75		
6	Interaction *B* × *C*	44.00	42.75	–1.25		
7	Factor *E*	43.00	43.75	0.75		

Table 6-22. Pooled ANOVA—Example 6-5

COLUMN	FACTOR NAMES	*f*	*S*	*V*	*F*	*P*
1	Factor *A*	(1)	(0.38)	Pooled		
2	Factor *C*	1	108.375	108.375	2.728	2.67
3	Interaction *A* × *C*	(1)	(9.380)	Pooled		
4	Factor *B*	1	1395.375	1395.375	35.126	52.72
5	Factor *D*	1	273.375	273.375	6.882	9.09
6	Interaction *B* × *C*	(1)	(9.380)	Pooled		
7	Factor *E*	(1)	(3.380)	Pooled		
All other/error		20	794.500	39.72		35.53
Total		23	2571.630			100.00%

$$S_B = \frac{(B_1 - B_2)^2}{(N_{B_1} + N_{B_2})} = \frac{(429 - 612)^2}{24} \qquad = 1395.375$$
$$V_B = S_B / f_B \qquad = 1395.375/1 \qquad = 1395.375$$
$$F_B = V_B / V_e \qquad = 1395.375/39.72 \qquad = 35.126$$
$$S'_B = S_B - V_e \times f_B \quad = 1395.375 - 39.72 \qquad = 1355.65$$
$$P_B = 100 \times S'_B / S_T = 100 \times 1355.655/2571.63 = 52.70\%$$

The ANOVA and the performance at the optimum condition are as shown in Tables 6-22 and 6-23, respectively.

When trial runs are repeated, ANOVA produces different results with larger error DOF and thus a higher level of confidence in the estimate optimum performance and the factor influences. ANOVA results from multiple repetitions should always be preferred as this will yield the robust design and reproducible performance estimate. For this reason, repetition is highly desirable. But because repeating trial runs may be expensive, it must be weighed against the need for robustness.

APPLICATION OF S/N RATIO

The change in the quality characteristics of a product under investigation in response to a factor introduced in the experimental design is the "signal" of the desired effect. However, when an experiment is conducted, there are numerous external and internal

Table 6-23. Estimate of performance at optimum condition—Example 6-5

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Factor *C*	C_2	2	–2.1250
Factor *B*	B_1	1	–7.6250
Factor *D*	D_2	2	–3.3750
Contribution from all factors (total)			–13.125
Current grand average of performance			43.375
Expected result at optimum condition			30.250

factors not designed into the experiment that influence the outcome. These uncontrollable factors are called the noise factors, and their effect on the outcome of the quality characteristic under test is termed "noise." The signal-to-noise ratio (S/N ratio) measures the sensitivity of the quality characteristic being investigated in a controlled manner to those influencing factors (noise factors) not under control. The concept of S/N originated in the electrical engineering field. Taguchi effectively applied this concept to establish the optimum condition from the experiments.

The aim of any experiment is always to determine the highest possible S/N ratio for the result. A high value of S/N implies that the signal is much higher than the random effects of the noise factors. Product design or process operation consistent with highest S/N always yields the optimum quality with minimum variance.

From the quality point of view, there are three typical categories of quality characteristics:

1. Smaller is better; for example, minimum shrinkage in a cast iron cylinder block casting.
2. Nominal is best; for example, dimension of a part consistently achieved with modest variance.
3. Bigger is better; for example, maximum expected life of a component.

The S/N analysis is designed to measure quality characteristics.

Conversion of Results into S/N Ratios

The conversion of a set of observations into a single number, the S/N ratio, is performed in two steps. First, the mean square deviation (MSD) of the set is calculated. Second, the S/N ratio is computed from the MSD by the equation,

$$\mathrm{S/N} = -10\log_{10}\left(\mathrm{MSD}\right) \tag{6-16}$$

Note that for the S/N to be large, the MSD must have a value that is small.

The smaller is better quality characteristic:

$$\mathrm{MSD} = \left(Y_1^2 + Y_2^2 + \ldots + Y_N^2\right)/N \tag{6-17}$$

The nominal is the best quality characteristic, Y_0 = nominal or target value:

$$\mathrm{MSD} = \left(\left(Y_1 - Y_0\right)^2 + \left(Y_2 - Y_0\right)^2 + \ldots + \left(Y_N - Y_0\right)^2\right) \Big/ N \qquad (6\text{-}18)$$

The bigger is better quality characteristic:

$$\mathrm{MSD} = \left(1/Y_1^2 + 1/Y_2^2 + \ldots + 1/Y_N^2\right) \Big/ N \qquad (6\text{-}19)$$

The MSD is a statistical quantity that reflects the deviation from the target value. The expressions for MSD are different for different quality characteristics. For the nominal is best characteristic, the standard definition of MSD is used. For the other two characteristics, the definition is slightly modified. For smaller is better, the unstated target value is zero. For larger is better, the inverse of each large value becomes a small value and, again, the unstated target is zero. Thus, for all three MSD expressions, the smallest magnitude of MSD is being sought. In turn, this yields the greatest discrimination between controlled and uncontrolled factors. This is Taguchi's measure for robust product or process design.

Alternate forms of definitions of the S/N ratios exist ([6], pp. 172-173), particularly for the nominal is best characteristic. The definition in terms of MSD is preferred as it is consistent with Taguchi's objective of reducing variation around the target. Conversion to S/N ratio can be viewed as a scale transformation for convenience of better data manipulation.

Advantage of S/N Ratio Over Average

To analyze the results of experiments involving multiple runs, use of the S/N ratio over standard analysis (use average of results) is preferred. Analysis using the S/N ratio will offer the following advantages:

1. It provides a guidance to selection of the optimum level based on the least variation around the target and also on the average value closest to the target.
2. It offers objective comparison of two sets of experimental data with respect to variation around the target and the deviation of the average from the target value.

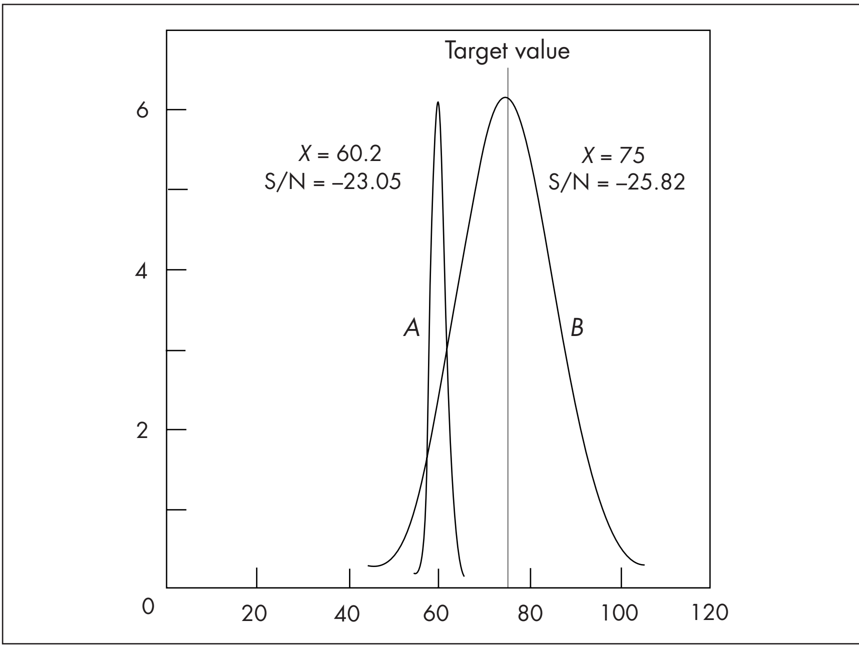


Figure 6-3. Comparison of two distributions

3. Because S/N represents results transformed into a logarithmic scale that linearizes any nonlinear behavior, if present, the assumption of linearity for prediction of optimum performance is validated.

To examine how the S/N ratio is used in analysis, consider the following two sets of observations, which have a target value of 75:

Observation A: 55 58 60 63 65	Mean	= 60.2
Deviation of mean from target		= (75 − 60.2)
		= 14.8
Observation B: 50 60 75 90 100	Average	= 75.00
Deviation of mean from target		= (75 − 75)
		= 0.0

These two sets of observations may have come from the two distributions shown in Figure 6-3. Observe that set B has an average value that equals the target value, but it has a wide spread around it. On the other hand, for set A, the spread around its average is smaller, but the average itself is quite far from the target. Which one of the two is better? Based on average value, the product shown by observation B appears to be better. Based on consistency, product A is better. How can one credit A for less variation? How does one compare the distances of the averages from the target? Surely, comparing the averages is one method. Use of the S/N ratio offers an objective way to look at the two characteristics together.

Computation of S/N Ratio

Consider the first of the two sets of observations shown above, that is, set A: 55 58 60 63 65.

Case 1. Nominal is best.
Using Eq. (6-18), and with the target value of 75,

$$\begin{aligned}\text{MSD} &= \left(\begin{matrix}(55-75)^2+(58-75)^2+(60-75)^2 \\ +(63-75)^2+(65-75)^2\end{matrix}\right)\Big/5 \\ &= (400+289+225+144+100)/5 \\ &= 1158/5 \\ &= 231.6\end{aligned}$$

therefore,

$$\begin{aligned}\text{S/N} &= -10\log(\text{MSD}) \\ &= -10\log(231.6) \\ &= -23.65\end{aligned}$$

Case 2. Smaller is better.
Using Eq. (6-17),

$$\begin{aligned}\text{MSD} &= \left(55^2+58^2+60^2+63^2+65^2\right)/5 \\ &= (3025+3364+3600+3969+4425)/5 \\ &= 18183/5 \\ &= 3636.6\end{aligned}$$

and $\text{S/N} = -10\log(3636.6)$
$$= -35.607$$

Case 3. Bigger is better.

Using Eq. (6-19),

$$\text{MSD} = \left(1/55^2 + 1/58^2 + 1/60^2 + 1/63^2 + 1/65^2\right)/5$$
$$= (1/3025 + 1/3364 + 1/3600 + 1/3969 + 1/4425)/5$$
$$= (3.305 + 2.972 + 2.777 + 2.519 + 2.366) \times 10^{-4}/5$$
$$= (13.939) \times 10^{-4}/5$$
$$= .0002787$$

therefore,

$$\text{S/N} = -10\log(.0002787)$$
$$= 35.548$$

The three S/N ratios computed for data sets A and B under the three different quality characteristics are shown in Table 6-24. The columns N, S, and B, under the heading "S/N ratios," are for nominal, smaller, and bigger characteristics, respectively.

Now select the best data set on the basis of minimum variation. By definition, lower deviation is indicated by a higher value of the S/N ratio (regardless of the characteristics of quality). If the nominal is best characteristic applies, then using column N, the S/N ratio for A is −23.65 and for B is −25.32. Because −23.65 is greater than −25.32, set A has less variation than set B, although set B has an average value equal to the desired target value.

Similarly, set A is selected for the smaller is better characteristic, and B is selected for the bigger is better characteristic.

Table 6-24. S/N ratios for three quality characteristics

		S/N RATIOS		
OBSERVATIONS	AVERAGE	*N*	*S*	*B*
Set A: 55 58 60 63 65	60.2	–23.65	–35.60	35.54
Set B: 50 60 75 90 100	75.0	–25.32	–37.76	36.65

Effect of S/N Ratio on the Analysis

Use of the S/N ratio of the results, instead of the average values, introduces some minor changes in the analysis.

- Degrees of freedom of the entire experiment is reduced.

DOF with S/N ratio = number of trial conditions − 1 (that is, number of repetitions is reduced to 1).

Recall that the DOF in the case of the standard analysis is:

DOF = (number of trials × number of repetitions) − 1

The S/N ratio calculation is based on data from all observations of a trial condition. The set of S/N ratios can then be considered as trial results without repetitions. Hence the DOF, in the case of S/N, is the number of trials − 1. The rest of the analysis follows the standard procedure.

- S/N must be converted back to meaningful terms. When the S/N ratio is used, the results of the analysis, such as estimated performance from the main effects or confidence interval, are expressed in terms of S/N. To express the analysis in terms of the experimental result, the ratio must be converted back to the original units of measurement.

To see the specific differences in the analysis using the S/N ratio, let us compare the two analyses of the same observations for the cam-lifter noise study shown in Table 6-25(a) (standard analysis) and in Table 6-25(b) (S/N ratio analysis). In this study, the three factors (spring rate, cam profile, and weight of the push rod), each at two levels, were investigated. The L_4 OA defined the four trial conditions. At each of the four trial conditions, three observations (in some noise scale of 0 to 60) were recorded. The results were then analyzed both ways, as shown in these two tables.

A subtable "Results" of the standard analysis [Table 6-25(a)] presents the average of the three repetitions for each trial run at the extreme right-hand column. The averages are used in calculating the main effects. The values shown in the subtable titled "Main effects" have the same units as the original observations. Similarly, the expected value at the optimum condition, 30.5,

Table 6-25(a). Cam lifter noise study—Standard analysis

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Spring rate	Current	Proposed		
2	Cam profile	Type 1	Type 2		
3	Wt. of push rod	Lighter	Heavier		

Individual Results and Their Average

REPETITION TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	23.00	30.00	37.00				30.00
2	35.00	40.00	45.00				40.00
3	50.00	30.00	40.00				40.00
4	45.00	48.00	51.00				48.00

Main Effects

COLUMN	FACTORS	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Spring rate	35.00	44.00	9.00	00.00	00.00
2	Cam profile	35.00	44.00	9.00	00.00	00.00
3	Wt. of push rod	39.00	40.00	1.00	00.00	00.00

ANOVA Table

COLUMN	FACTORS	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
1	Spring rate	1	243.00	243.00	5.93	23.63
2	Cam profile	1	243.00	243.00	5.93	23.63
3	Wt. of push rod	(1)	(3.00)	POOLED		
All other/error		9	369.00	41.00		52.76
Total:		11	855.00			100.00

Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Spring rate	Current	1	–4.50
Cam profile	Type 1	1	–4.50
Contribution from all factors (total)			–9.00
Current grand average of performance			39.50
Expected result at optimum condition			30.50

Table 6-25(b). Cam lifter noise study—S/N analysis

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Spring rate	Current	Proposed		
2	Cam profile	Type 1	Type 2		
3	Wt. of push rod	Lighter	Heavier		

Individual Results and S/N Ratios

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	S/N
1	23.00	30.00	37.00				–29.70
2	35.00	40.00	45.00				–32.09
3	50.00	30.00	40.00				–32.22
4	45.00	48.00	51.00				–33.64

Main Effects

COLUMN	FACTORS	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Spring rate	–30.9	–32.93	–2.04	00.00	00.00
2	Cam profile	–30.96	–32.86	–1.91	00.00	00.00
3	Wt. of push rod	–31.67	–32.16	–0.49	00.00	00.00

ANOVA Table

COLUMN	FACTORS	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	Spring rate	1	4.141	4.141	17.61	48.79
2	Cam profile	1	3.629	3.629	15.43	42.39
3	Wt. of push rod	(1)	(0.24)	POOLED		
All other/error		1	0.24	0.24		8.82
Total:		3	8.01			100.00

Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Spring rate	Current	1	1.0175
Cam profile	Type 1	1	0.9525
Contribution from all factors (total)			1.9699
Current grand average of performance			–31.9125
Expected result at optimum condition			–29.9425

has the same units as the original recorded data. The degrees of freedom for the experiment (DOF column in ANOVA table) is 11 ($4 \times 3 - 1$).

Comparing the standard analysis with the analysis using the S/N ratio [Table 6-25(b)], note that the average value of the results is replaced by the S/N ratio. The S/N ratios are then used to compute the main effects as well as the estimated performance at the optimum condition. Notice also that the degrees of freedom for the experiment is 3. This difference in DOF produces a big difference in the way the two analyses compute ANOVA, that is, the percentage contribution of the factors involved (for spring rate, the value is 23.6% from standard analysis as compared with 48.79% from S/N analysis). Likewise, the other factors will have different magnitudes of contribution in the two methods.

In estimating the result at the optimum condition, only the factors that will have significant contributions are included. In this case, both methods selected level 1 of factors in columns 1 (spring rate) and 2 (cam profile). This may not always be true.

When the S/N ratio is used, the estimated result can be converted back to the scale of units of the original observations. For example, the expected result in terms of S/N ratio is −29.9425 [Table 6-25(b), bottom line]. This is equivalent to an average performance, Y', which is calculated as follows:

Because

$$\mathrm{S/N} = -10 \log(\mathrm{MSD})$$

and

$$\mathrm{MSD} = \left(Y_1^2 + \ldots + Y_N^2\right)/N \quad (\text{for smaller is better})$$

$$\cong Y_{\text{expected}}^2$$

Therefore,

$$\mathrm{MSD} = 10^{(-\mathrm{S/N})/10} = 10^{-(-29.9425)/10} = 986.8474$$

or

$$Y_{\text{expected}} \cong (\mathrm{MSD})^{1/2} = (986.8474)^{1/2} = 31.41$$

which is comparable to 30.5 shown at the bottom of Table 6-25(a).

When to Use S/N Ratio for Analysis

Whenever an experiment involves repeated (two or more) observations at each of the trial conditions, the S/N ratio has been found to provide a practical way to measure and control the combined influence of deviation of the population mean from the target and the variation around the mean. In standard analysis, the mean and the variation around the mean are treated separately by a main effect study and ANOVA, respectively. The analysis of the Taguchi experiments using S/N ratios for the observed results can be conveniently performed by using computer software such as [7].

EXERCISES

6-1. In an experiment involving four factors (A, B, C, and D) and one interaction (A × B), each trial condition is repeated three times and the observations recorded as shown in Table 6-26. Determine the total sum of squares and the sum of squares for factor A.

6-2. Assuming the "bigger is better" quality characteristic, transform the results of trial 1 (Table 6-26) into the corresponding S/N ratio.

6-3. Table 6-27 shows the product of ANOVA performed on the observed results of an experiment. Determine the following from the ANOVA table.
   a. Percent influence of the clearance factor.
   b. Degrees of freedom of the speed factor.
   c. Error degrees of freedom.
   d. Influence of noise factors and all other factors not included in the experiment.
   e. Confidence interval (90%) of the performance at the optimum condition (use *F*-table for 90% confidence level).

Table 6-26. Orthogonal array and test data

COLUMN TRIAL	*A* 1	*B* 2	$A \times B$ 3	*C* 4	*D* 5	6	7	R_1	R_2	R_3
Trial 1	1	1	1	1	1	0	0	45.00	56.00	64.00
Trial 2	1	1	1	2	2	0	0	34.00	45.00	53.00
Trial 3	1	2	2	1	1	0	0	67.00	65.00	60.00
Trial 4	1	2	2	2	2	0	0	45.00	56.00	64.00
Trial 5	2	1	2	1	2	0	0	87.00	81.00	69.00
Trial 6	2	1	2	2	1	0	0	78.00	73.00	68.00
Trial 7	2	2	1	1	2	0	0	45.00	56.00	52.00
Trial 8	2	2	1	2	1	0	0	42.00	54.00	47.00

Table 6-27. ANOVA

COLUMN	FACTOR NAMES	*f*	*S*	*V*	*F*	*P*
1	Speed	1	3.036	3.036	4.432	9.56
2	Oil viscosity	(1)	(1.91)	Pooled		
	Interaction	1	15.820	15.820	23.093	61.57
4	Clearance	1	2.987	2.987	4.361	9.36
5	Pin straightness	(1)	(0.75)	Pooled		
All other/error		4	2.740	0.685		19.51
Total		7	24.584			100.00%

Note: Insignificant factorial effects are pooled as shown ( ).

7 Loss Function

DERIVATION OF LOSS FUNCTION

In Chapter 2, Taguchi's philosophy regarding the cost of quality was stressed. In his view, a poorly designed product causes society to incur losses from the initial design stage through to product usage. Therefore, he emphasizes good quality at the conceptual stage of a product and onward, by optimizing the product design parameters as well as the production conditions, to create a robust product.

A question commonly asked is how much effort should an organization expend on quality. How can the point of diminishing returns be determined? In this section the basic mathematical formulation of Taguchi's loss function is developed, and an outline of the steps used to apply the loss function is presented. The loss function has proven to be an excellent tool for determining the magnitude of the process (manufacturer) and supplier tolerances, based on quality as perceived by the customer. The methodology for realignment of tolerances is beyond the scope of this text.

Taguchi defined the loss function as deviation as a quantity proportional to the deviation from the target quality characteristic. At zero deviation, the performance is on target and the loss is zero. If Y represents the deviation from the target value, then the loss function $L(Y)$ is:

$$L(Y) = k(Y - Y_0)^2 \tag{7-1}$$

where

Y = quality characteristics, such as dimension, performance, and so on

Y_0 = target value for the quality characteristic
k = a constant, dependent on the cost structure of a manufacturing process or an organization

It is important to note that:

1. The term $(Y - Y_0)$ represents the deviation of the quality characteristic Y from the target value Y_0.
2. The equation for the loss function is of the second order in terms of deviation of the quality characteristic.

The loss function represented by Eq. (7-1) is graphically shown in Figure 7-1; it possesses the following characteristics:

1. The loss must be zero when the quality characteristic of a product meets its target value.
2. The magnitude of the loss increases rapidly as the quality characteristic deviates from the target value.

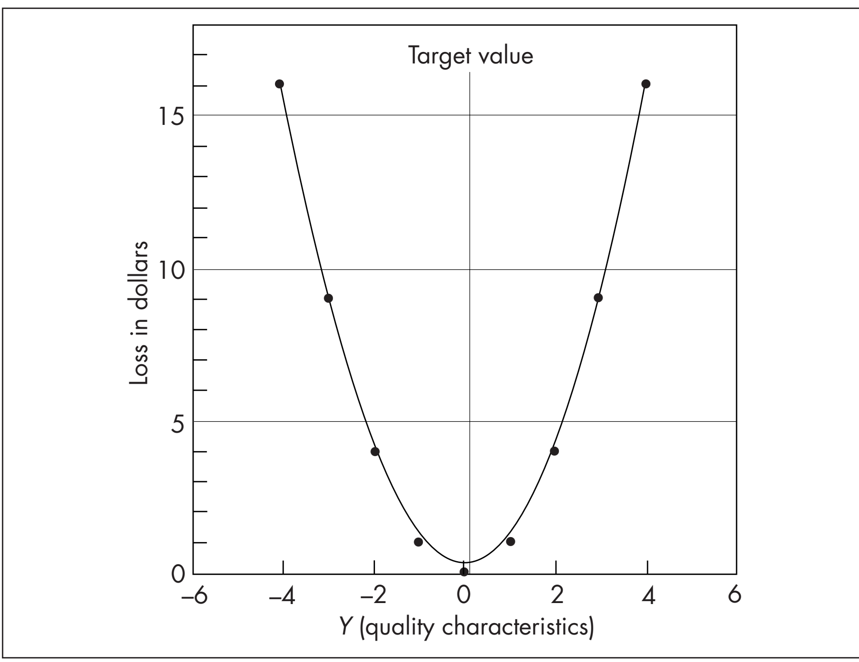


Figure 7-1. Taguchi quality loss function

3. The loss function must be a continuous (second-order) function of the deviation from the target value.

Taguchi determined the loss function from a Taylor's theorem expansion about the target value, Y_0. Thus,

$$L(Y) = L(Y_0) + L'(Y_0)(Y - Y_0) + \frac{1}{2!}L''(Y_0)(Y - Y_0)^2 \qquad (7\text{-}2)$$

where the terms with powers of $(Y - Y_0)$ higher than 2 are ignored as being too small for consideration.

In Eq. (7-2), $L(Y)$ is the minimum at $Y = Y_0$; hence, its first derivative, $L'(Y_0)$, is zero. Therefore, Eq. (7-2) can be written as

$$L(Y) = L(Y_0) + \frac{1}{2!}L''(Y_0)(Y - Y_0)^2 \qquad (7\text{-}3)$$

The expression $(1/2)L''(Y_0)$ in Eq. (7-3) is a constant and can be replaced by a constant k. Also, if Y_0 is the mean value of the product/process, then Eq. (7-3) is interpreted as the loss about the product/process mean plus the loss due to displacement of the process mean from the target. If the process mean coincides with the target, the loss term $L(Y_0)$ is zero and the loss function reduces to Eq. (7-1),

$$L_0 = k(Y_0 + \Delta - Y_0)^2$$

The magnitude of the loss incurred because of the inability of a process to meet the target value of a quality characteristic, as computed using Eq. (7-1), is dependent on the target value, the manufacturing process, and the cost of rework, scrap, and warranty. For a given value of Y_0, the value of k varies with the process and the organization. An organization seriously committed to achieving higher standards of quality in an optimum manner may develop families of loss curves for each process. One family of curves with different values of k is shown in Figure 7-2. The value of k for any application can be determined as outlined below.

Any mass-produced product exhibits variation in its quality characteristic. As long as the variation is small, the quality of the product is acceptable to the customer. Customer acceptance

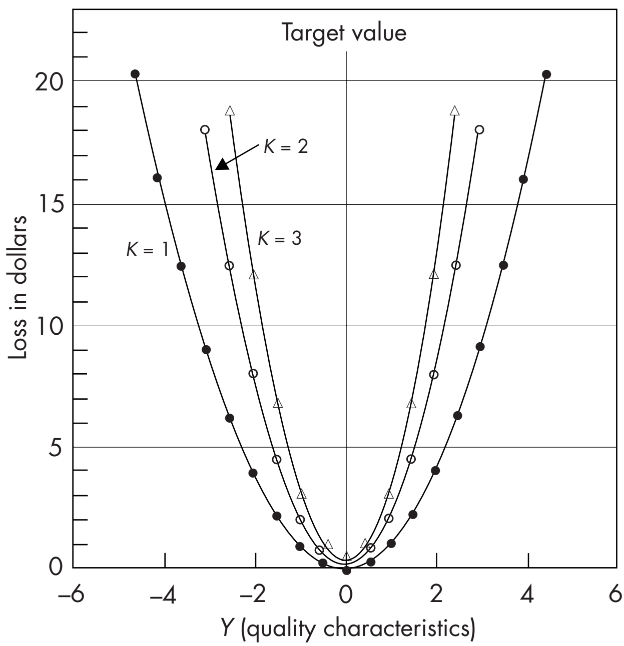


Figure 7-2. Family of loss functions

determines the range of variation. If the perceived quality falls outside the range, the customer will not accept the product, and corrective actions at the design or process level are required.

Let this tolerance zone be $\pm\Delta$. Then the quality characteristic at the extremes can be denoted as:

$Y_0 + \Delta$ and $Y_0 - \Delta$

Poor quality exceeding the extremes necessitates corrective action such as a warranty cost of L_0.

Let L_0 be the loss at $Y = Y_0 + \Delta$

Then from Eq. (7-1):

$$L_0 = k\left(Y_0 + \Delta - Y_0\right)^2$$

or $$k = \frac{L_0}{\Delta^2}$$

Therefore, Eq. (7-1) for this case becomes:

$$L(Y) = \frac{L_0}{\Delta^2}(Y - Y_0)^2$$

The above equation now completely defines the loss function in terms of the deviation from the target value.

AVERAGE LOSS FUNCTION FOR PRODUCT POPULATION

The loss function given in Eq. (7-1) represents the financial loss experienced by a single product when the quality characteristic, Y, of the product deviates from the target value, Y_0. In a mass-production process, the average loss per unit is expressed by:

$$L(Y) = \left[k(Y_1 - Y_0)^2 + k(Y_2 - Y_0)^2 + k(Y_3 - Y_0)^2 + \ldots + k(Y_n - Y_0)^2\right]\Big/n$$

where n is the number of units in a given sample.

In the above equation, the factor k is common with every term, and therefore it can also be written as:

$$L(Y) = k\left[(Y_1 - Y_0)^2 + (Y_2 - Y_0)^2 + (Y_3 - Y_0)^2 + \ldots + (Y_n - Y_0)^2\right]\Big/n$$

Note that the expression within the brackets is the mean square deviation (MSD), the average of the squares of all deviations from the target value, Y_0. The average loss per unit can now simply be expressed by:

$$L(Y) = k(\text{MSD}) \tag{7-4}$$

APPLICATION OF LOSS FUNCTION CONCEPTS

The loss function concept has two practical applications. The primary application is for estimating the potential cost savings resulting from the improvements achieved by optimizing a product or process design. The loss function can serve as a measure of performance regardless of the method of the quality improvement. As long as variation is reduced by corrective actions or design improvements, the loss function presents a means for estimating the savings in terms of dollars and cents. It can also be used to determine if an investment to reduce variation is worth the cost. The second application is to determine manufacturer and supplier tolerances based on the customer's perception of the quality range. In this case, the loss function provides an objective way to set the limits for the inspection of products at the manufacturer or supplier location. The following examples illustrate the use of the loss function.

Example 7-1
Machine Bracket Casting Process (Cost Savings)

Engineers involved in casting a machine bracket designed an experimental study to improve the process and reduce scrap rate. As a result of the study, a number of improvements were incorporated. Data were taken from 10 samples before and after the experiments. The foundry had a production rate of 1500 castings per month. The quality inspection criterion was a length dimension of 12 ± 0.35 inches. The parts that did not fall within the limits were rejected. The average unit cost for scrap or rework of the rejects was \$20. The potential cost savings of the optimized process was calculated by the Taguchi loss function.

TEST DATA

Before Experiment

11.80	12.30	12.20	12.4	12.1	12.2	11.9	11.8	11.85	12.15

After Experiment

11.9	12.2	12.1	12.2	12.1	12.1	11.9	11.95	11.95	12.1

Other Data:

Target value	=	12.00 in.
Tolerance	=	±0.35 in.
Cost of rejection	=	\$20.00
Production rate	=	1500 per month

Solution

For this application, the expression of loss in terms of the MSD will be used.

$L(Y) = k(Y - Y_0)^2$ for a single sample and

$L(Y) = k(\text{MSD})$ for multiple samples

Using Eq. (7-1), the constant k is determined as follows:

$L = k(Y - Y_0)^2$

When all parts are made just outside of the specifications, that is, when

$Y = Y_0 \pm \text{Tolerance},$

then $L = k(Y_0 \pm \text{Tolerance} - Y_0)^2$

But the loss L in this case equals the cost of rejecting a part ($20.00), and the tolerance is 0.35.

or $20 = k(.35)^2$

or $k = 20/(.35)^2 = 163.265$

Therefore, from Eq. (7-4),

$$L = 163.265\ (\text{MSD}) \tag{7-5}$$

Using the data from samples before the experiment,

$$\text{MSD} = \left[(11.8 - 12)^2 + (12.3 - 12)^2 + \ldots\right]/10$$
$$= 0.0475$$

The MSD and other statistical parameters for this example, as shown in Tables 7-1, 7-2, and 7-3, are obtained by using the software in [7]. The format of the design descriptions and the results are presented in the manner displayed by the software.

From Eq. (7-5) the average loss per unit is calculated as:

$L = 163.265 \times .0475 = 7.754$ (in dollars)

Using the data from samples after the experiment,

$$\text{MSD} = \left[(11.9 - 12)^2 + (12.2 - 12)^2 + \ldots\right]/10$$
$$= 0.0145$$

From Eq. (7-6), the average loss per unit is calculated to be:

$L = 163.265 \times .0145 = 2.367$ (in dollars)

Table 7-1. Machine bracket casting process (Before experiment)

Observation No. 1	=	11.800
Observation No. 2	=	12.300
Observation No. 3	=	12.200
Observation No. 4	=	12.400
Observation No. 5	=	12.100
Observation No. 6	=	12.200
Observation No. 7	=	11.900
Observation No. 8	=	11.800
Observation No. 9	=	11.850
Observation No. 10	=	12.150
Target/nominal value of result	=	12.00
Number of test results (NR)	=	10
AVERAGE AND STANDARD DEVIATION:		
Total of all test results	=	120.70000
Average of test results	=	12.07000
Standard deviation (SD)	=	00.21756
Variance	=	00.04733
LOSS FUNCTION PARAMETERS:		
Mean square deviation (MSD)	=	00.04749
Signal-to-noise (S/N) ratio	=	13.23307
Variance (modified form)	=	00.04259
Square of mean value	=	00.00489
VARIANCE DATA (ANOVA):		
Target value of data/test result	=	12.00
Mean of data/deviation from target	=	00.069999
Total variance (ST)	=	00.426
(ST = variance * NR)		
Correction factor (CF)	=	00.04899
(CF = (average of data)^2 * number of data)		
Sums of squares/*N*	=	00.47499

The average savings per unit is calculated by subtracting the loss after the experiment ($2.367) from that before the experiment ($7.754). The total savings is then obtained by multiplying the average savings by the production rate as shown here.

$$\text{Total savings per month} = (7.754 - 2.367) \times 1500 = \$8080.50$$

Table 7-2. Machine bracket casting process (After experiment)

Observation No. 1	=	11.900
Observation No. 2	=	12.200
Observation No. 3	=	12.100
Observation No. 4	=	12.200
Observation No. 5	=	12.100
Observation No. 6	=	12.100
Observation No. 7	=	11.900
Observation No. 8	=	11.950
Observation No. 9	=	11.950
Observation No. 10	=	12.100
Target/nominal value of result	=	12.00
Number of test results (NR)	=	10
AVERAGE AND STANDARD DEVIATION:		
Total of all test results	=	120.50000
Average of test results	=	12.05000
Standard deviation (SD)	=	00.11547
Variance	=	00.01333
LOSS FUNCTION PARAMETERS:		
Mean square deviation (MSD)	=	00.01450
Signal-to-noise (S/N) ratio	=	18.38631
Variance (modified form)	=	00.01200
Square of mean value	=	00.00250
VARIANCE DATA (ANOVA):		
Target value of data/test result	=	12.00
Mean of data/deviation from target	=	00.05000
Total variance (ST)	=	00.12000
(ST = variance * NR)		
Correction factor (CF)	=	00.02500
(CF = (average of data)^2 * number of data)		
Sums of squares/*N*	=	00.14500

Example 7-2
Dryer Motor Belt (Manufacturer/Supplier Tolerance)

Alarmed by a high rate of warranty repairs of drive belts for one of its products, the distributor sought to reduce such defects. The field reports suggested that the problem was mainly caused by the lack of adjustment of tension in the drive belt. To correct

Table 7-3. Calculation of loss

PROBLEM DEFINITION		
Target value of quality characteristic (*m*)	=	12.00
Tolerance of quality characteristic	=	0.35
Cost of rejection at production (per unit)	=	\$20.00
Units produced per month (total)	=	1500
S/N ratio of current design/part	=	13.23307
S/N ratio of new design/part	=	18.38631
COMPUTATION OF LOSS USING TAGUCHI LOSS FUNCTION		
Loss function: $L(y) = 163.26 \times (\text{MSD})$ Also $L(y)$	=	$K \times (y - m)^2$
BEFORE EXPERIMENT:		
Loss/unit due to deviation from target in current design	=	\$7.754
AFTER EXPERIMENT:		
Loss/unit due to deviation from target will be reduced from \$7.754 to	=	\$2.367
MONTHLY SAVINGS:		
If production is maintained at the improved condition, then based on 1500 units/month	=	\$8080.50

the situation at the customer's location, the field repairmen had to adjust the tension to 100 ± 15 lbs. Field service cost is \$40 per unit. Alternately, the adjustment of tension could be made by the manufacturer at a unit cost of \$15. The distributor wants to ask the manufacturer to make such adjustments prior to shipment in order to eliminate the field service and maintain satisfied customers. What range of tolerance in belt tension should the distributor specify for the manufacturer?

Solution

For this application, an understanding of the role of the three parties, namely customer, manufacturer, and supplier, will be helpful. In the context of tolerance specification, the three terms correspond to three stages of product life. The supplier is the one who supplies a component or part of the finished product to the manufacturer. The manufacturer is the one who assembles the final product. The customer is the one who uses the product and experiences its performance. In this example, the distributor and the customers are considered to be the end users. The customer and the manufacturer may have a supplier (not identified) for the motor and belt assembly. The relationships among the three can be represented in the following way.

Supplier	→	Manufacturer	→	Customer
(Belt + motor)		(Washer)		(Washer in use)
Tolerance required:				
(unknown)		(unknown)		(±15 lbs.)

From Eq. (7-1) we have

$$L(Y) = k(Y - Y_0)^2 = k\ (\text{Tolerance})^2$$

where

$$\text{Tolerance} = Y(\text{max. or min.}) - Y_0$$

Based on a repair cost of \$40 at the customer's installation, the loss per unit is \$40.

Because

$$Y_0 = 100,\ Y = 100 + 15\ (\text{max.}),\ \text{and}\ L(Y) = 40$$

$$k = 40/(100 + 15 - 100)^2 = 0.17778$$

Therefore,

$$L(Y) = .17778\ (\text{Tolerance})^2 \qquad (7\text{-}6)$$

Using the repair cost of \$15 at the manufacturer's facility as the loss, the tolerance now can be determined by using the above relation.

$$\underset{(\text{Manufacturer})}{\text{Tolerance}} = (L/.17778)^{1/2} = 9.18$$

or

$$\text{Tolerance limits} = 100 \pm 9.8$$

If the manufacturer determined the problem to be caused by a specific component, then correction at the supplier's facilities would be appropriate. A different set of specifications may be required. Assuming the cost to make an adjustment at the supplier is \$5 per unit, what tolerance should the manufacturer require of the supplier to assure the required quality?

Using Eq. (7-4) with $L(Y) = 5$

$$5 = .178\ (\text{Tolerance})^2$$

and

$$\text{Tolerance} = (5/.17778)^{1/2} = 5.30$$

Therefore

$$\text{Tolerance limits for supplier} = 100 \pm 5.3$$

The solutions are presented in Figure 7-3.

Example 7-3
Fuel Pump Noise Study

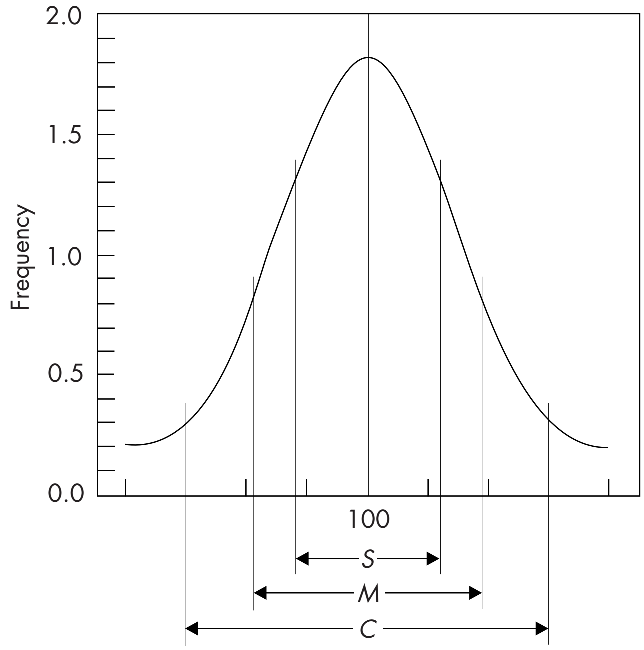
In an experimental study of an automotive fuel pump noise, three two-level factors were included as shown in Table 7-4(a). The Taguchi L_4 orthogonal array was used to define the four trial conditions. Six samples at each of the trial conditions were tested, and the results were recorded as shown in Table 7-4(b). The levels were selected so that trial condition 1 represents the current design of the fuel pump. If the decision is made to change the design to the determined optimum configuration, estimate the performance at the optimum design and the cost savings when the new fuel pump is produced.

Solution

The complete solution of this problem is shown in Tables 7-4(b) and 7-4(c). In calculating the cost savings, notice that the S/N ratios at trial condition 1 and the optimum condition are taken directly from the analysis of the Taguchi experimental results. (Solutions used [7].)

LOOKS OF PERFORMANCE IMPROVEMENT

The common purpose for carrying out an experimental study is to determine a new design condition (improved) that is better



S = Supplier tolerance 100 ± 5.8
M = Manufacturer tolerance 100 ± 9.18
C = Customer tolerance 100 ± 15

COMPUTATION OF MANUFACTURER AND SUPPLIER TOLERANCE

Nominal value of the quality characteristic	*Y* = 100 lbs
Tolerance of Y (range of deviation)	= ±15 lbs
Cost to repair a nonfunctioning unit by customer	= $40.0
Cost to repair a nonfunctioning unit by manufacturer	= $15.0
Cost to repair a nonfunctioning unit by supplier	= $ 5.0

REQUIRED TOLERANCES

Manufacturer tolerance = 100 ± 9.18
Supplier tolerance = 100 ± 5.3

NOTE: If these tolerances are held, there will be no nonfunctional part in the customer's hands. For the same cost, the manufacturer will maintain satisfied customers and quality products in the field.

Figure 7-3. Manufacturer and supplier tolerances

Table 7-4(a). Fuel pump noise study—Example 7-3

COLUMN	FACTOR	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Seal thickness	Present	Thicker		
2	Rotor chuck type	Present	New design		
3	Finger to drive C_1	Present	Increase		

Note: Three two-level factors studied.
Objective: Design least noisy and best-performing pump.
Characteristic: Nominal is best (SIQ = 60 target).

This experiment will use L_4.

COLUMN / TRIAL	1	2	3
Trial 1	1	1	1
Trial 2	1	2	2
Trial 3	2	1	2
Trial 4	2	2	1

than the current status. When improvement is achieved, it is necessarily reflected in lowering the standard deviation (variance, σ^2) and/or reducing the distance of mean performance from the target. Of course, when variation is reduced, with or without change in distance to the target, common performance measures like capability indices (C_p and C_{pk}) increase and Taguchi loss (L) decreases. While all of these numerical indices are easily computed, for better visualization of the improvement a plot of the distribution is most desirable.

A direct way to draw a distribution diagram is possible when a large number of data (N) is available. Unfortunately, test sample size in DOE is generally small. In this case and in situations where the observed performance data are not available, the distribution can only be plotted from analytical expressions. When performance is assumed to be normal, the distribution plot can be easily created using the average and the standard deviation for the expected performance.

Table 7-4(b). Fuel pump noise study (Result: Main effect and ANOVA)

Original Observations and Their S/N Ratios
Quality Characteristic: Nominal is Best

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	S/N
1	67.00	85.00	87.00	65.00	59.00	76.00	–20.71
2	65.00	65.00	66.00	54.00	73.00	58.00	–18.99
3	54.00	45.00	56.00	45.00	63.00	46.00	–25.89
4	56.00	67.00	45.00	54.00	56.00	74.00	–23.36

Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Seal thickness	–19.85	–24.63	–4.78	00.00	00.00
2	Rotor chuck type	–23.30	–21.18	2.12	00.00	00.00
3	Finger to drive	–22.04	–22.44	–0.41	00.00	00.00

ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	Seal thickness	1	22.801	22.801		82.97
2	Rotor chuck type	1	4.512	4.512		16.43
3	Finger to drive	1	0.164	0.164		00.60
All other/error		0				
Total:		3	27.480			100.00

Regardless of the nature of distribution, given a set of observed data (say, 9.2, 8.9, 9.3, 9.6, and so on, for a 9-volt battery sample), average (a), σ, MSD, S/N, C_p, and C_{pk} can be easily calculated. Conversely, if S/N is known, as is the case when DOE results are analyzed using S/N ratios, value of the expected σ can be estimated.

The current condition of a product studied for improvement was found to have the following statistics.

Current Condition

Average performance aligned with the target value (assumed for simplistic calculation) is:

Table 7-4(c). Fuel pump noise study (Optimum and cost savings)

Estimate of Optimum Condition of Design/Process
Quality Characteristic: Nominal is Best

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Seal thickness	Present design	1	2.3875
Rotor chuck type	New design	2	1.0625
Finger to drive clearance	Present design	1	0.2025
Contribution from all factors (total)			3.6524
Current grand average of performance			–22.2375
Expected result at optimum condition			–18.5850

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

CALCULATION OF LOSS

PROBLEM DEFINITION

Target value of quality characteristic (m)	=	70.00
Tolerance of quality characteristic	=	20.00
Cost of rejection at production (per unit)	=	\$45.00
Units produced per month (total)	=	20000
S/N ratio of current design/part	=	–20.71
S/N ratio of new design/part	=	–18.585

COMPUTATION OF LOSS USING
TAGUCHI LOSS FUNCTION
Loss function: $L(y) = 0.11 \times (\text{MSD})$ Also $L(y) = K \times (y - m)^2$

BEFORE EXPERIMENT:
Loss/unit due to deviation from target
in current design = \$12.953

AFTER EXPERIMENT:
Loss/unit due to deviation from target
will be reduced from \$12.953 to = \$7.941

MONTHLY SAVINGS:
If production is maintained at the improved
condition, then based on 20000 units/month = \$100,246.90

S/N = –35.249 (or MSD = 3348.88) and
Std. dev. (σ) = 13.402

The specification limits needed for calculation of capability statistics are:

Lower specification limit (LSL) = 18.374
Upper specification limit (USL) = 98.791

Improved Condition

After completing the experimental study for the *smaller is better* quality characteristic, the performance at optimum condition (improved condition) expressed in S/N ratio was estimated to be:

S/N = 32.081 (or MSD = 1614.73)

Estimated Statistics at Improved Condition

Based on the known statistics at the current condition and S/N at the improved condition, the expected performance with the improved design can be calculated and the distribution represented as shown below.

Because MSD α σ^2 (when mean performance is on target, MSD is proportional to variance), then

$$\sigma^2(\text{improved}) = \frac{\text{MSD}_{\text{current}}}{\text{MSD}_{\text{improved}}} \times \sigma^2(\text{current})$$

or

$$\sigma^2(\text{improved}) = \frac{1614.73}{3348.88} \times 13.402^2$$
$$= 9.306$$

Because Loss (L) α MSD, then

$$L(\text{improved}) = \frac{\text{MSD}_{\text{current}}}{\text{MSD}_{\text{improved}}} \times L(\text{current})$$
$$= \frac{1614.73}{3348.88} \times \$1.00 \begin{pmatrix}\text{assumed loss in} \\ \text{current condition}\end{pmatrix}$$
$$= \$0.483$$

Thus, Savings (1.00 – 0.483) = 51.7 cents for every dollar spent at current condition.

And because Process Capabilities (C_p and C_{pk}) α ($1/\sigma$)

$$C_p = \frac{\sigma_{\text{current}}}{\sigma_{\text{improved}}} \times C_p\,(\text{current})$$

and

$$C_{pk} = \frac{\sigma_{\text{current}}}{\sigma_{\text{improved}}} \times C_{pk}\,(\text{current})$$

that is

$$C_p \text{ and } C_{pk} = \frac{13.402}{9.306} \times 1$$
$$= 1.44$$

The plot of variation reduced by adopting optimum design along with statistics calculated above is shown in Figure 7-4 (graph from [7]). The reduction of variation is expected to lower the rejection and warranty items, which results in cost savings expressed in terms of percentage of the loss at the current condition. A single figure like this can capture the essence of improvement expected and represent it graphically for all to understand.

EXERCISES

7-1. The manufacturer of a 10.5-volt smoke alarm battery employed the Taguchi method to determine the better design parameters. The experimenters estimated the signal-to-noise (S/N) ratio for the proposed design to be 6.3. Based on a sample inspection of the current production process, the S/N ratio was calculated to be 5.2. The analysis of warranty showed that when the battery voltage was beyond (10.50 ± 0.75) volts, the smoke alarm malfunctioned and customers returned the batteries for \$6.50 each. Determine the monthly savings that the proposed new design is expected to generate if 20,000 units are manufactured each month.

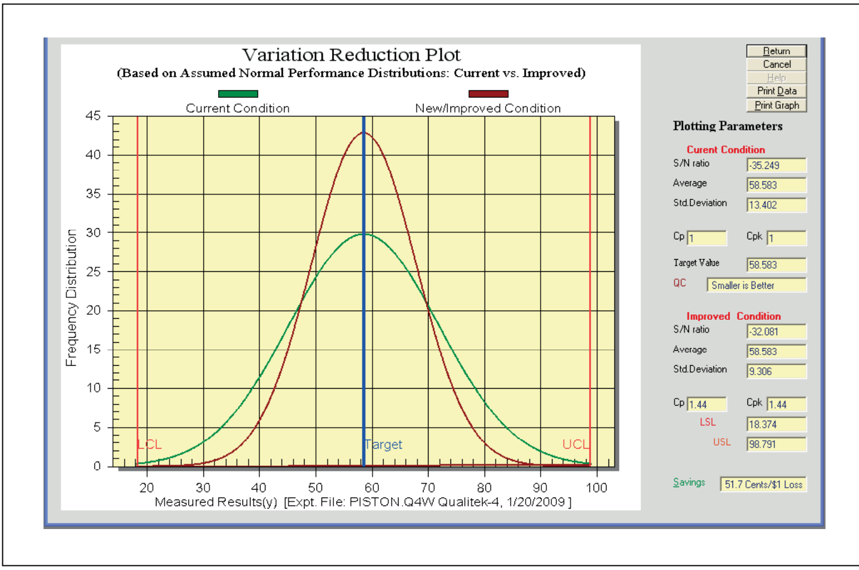


Figure 7-4. Plot of variation reduction as indication of performance improvement (graph from [7])

7-2. Suppose that the manufacturer in Exercise 7-1 decides not to adopt the new design but chooses to screen all defective batteries in the manufacturing plant before they are shipped to the customers. The cost for inspection in the plant is estimated to be $3 per battery. If the same amount of warranty cost is incurred in the inspection process, determine the tolerance limits for the inspection.

8 Brainstorming—An Integral Part of the Taguchi Philosophy

THE NECESSITY OF BRAINSTORMING

In applying the Taguchi technique, brainstorming is a necessary step in the process (Figure 8-1) and is essential for designing effective experiments. Taguchi recommends brainstorming to overcome cross-organizational barriers. By including representatives of all departments in the project team, from design through marketing, the quality demanded by the customer can be considered, and those production factors that may contribute toward quality can be identified and incorporated into the design of the experiments.

The benefits of brainstorming are obvious. The design does not belong to any one group—it belongs to all. Brainstorming identifies characteristic effects and the environment known to the group as a whole. Measurement techniques can draw on many disciplines. The outcome is better than it would be when one activity assumed all responsibilities.

Taguchi does not prescribe a standard method of brainstorming as applicable to all situations. The nature and content of the brainstorming session will vary widely depending on the problem and the experience of practitioners. For most studies, a formal session is highly effective. In many instances, brainstorming can be completed in a few hours, but for most projects the session may take a good part of a day dedicated for the purpose.

One method of conducting brainstorming, or what is normally referred to as an experiment planning session for the Taguchi experimental design, is presented in the text. The procedure has been found to be effective by the author in his experience with

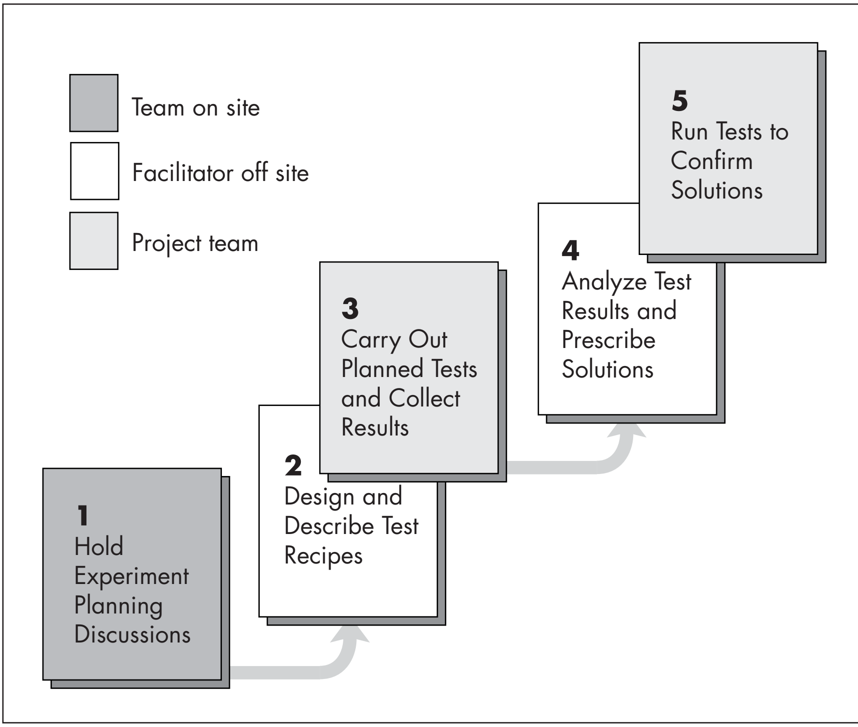


Figure 8-1. Five-step product or process study roadmap

various client industries. The procedure is not standard nor is it Taguchi's; it is the author's.

THE NATURE OF THE SESSION

The following guidelines help determine the participants and the content of the brainstorming session:

Purpose of the Brainstorming or Experiment Planning Session

1. Determine project objectives; identify factors, levels, and other pertinent information about the experiment,

collectively, with all of the project team composed of personnel from departments concerned with a successful outcome from the experiment.

2. Build team spirit and attitude to assure maximum participation and ownership of the team members.
3. Develop a consensus on the selection and the determination of those items that are objective and those that are subjective in nature.

Team Leader

For the successful completion of a Taguchi case study, the appointment of a team leader, from among the project team members, is necessary. The team leader must recognize the need for a brainstorming session and call for such a session. The leader should try to hold the session on neutral ground on a pre-announced day. The leader should ensure the participation of all team members with responsibilities for the product/process.

Session Facilitator

The session should be facilitated by someone with a good working knowledge of the Taguchi methodologies. Engineers or statisticians dedicated to helping others apply this tool often make better facilitators. A facilitator need not be a participator unless the project leader facilitates the session. The facilitator initiates and leads the discussion but never dominates it.

Who Should Form the Project Team?

All those who have first-hand knowledge and/or a stake in the outcome should be included. For an engineering design or a manufacturing process, both the design and the manufacturing personnel should be part of the team. If cost or supplier knowledge are likely factors, then persons with experience in these matters should be encouraged to participate (group size permitting). Marketing personnel should attend the planning session to provide customer viewpoints.

What Should Be the Size of Project Team?

The more the better. However, the time involved is proportional to the number attending. The upper limit should be 15. There can be as few as two. No matter the number of people involved, brainstorming will immensely benefit the whole process. More important than size is proper representation from all departments involved in design, development, production, marketing, sales, and service.

Is Taguchi Training a Prerequisite?

No. Some exposure will help. Application experience on the part of some participants will be a plus. A facilitator with application experience can help the participants with brief overviews when needed.

What is the Agenda for the Session?

The experiment planning session is the first step in application process and is preceded by:

- Identification of a project by sponsors and stakeholders
- Appointment of the project leader and
- Formation of the project team

Below is the recommended list of topics and sequence of discussions as shown in Figure 8-2. The emphasis and length of the discussion, however, will differ significantly for different problems being addressed.

TOPICS OF THE DISCUSSIONS

The following topics should be included in the agenda for the brainstorming session.

- Define the system under study
- Select project title
- Describe objectives of experiment
- Define performance (results) and units of measurement
- Determine evaluation criteria and create OEC table

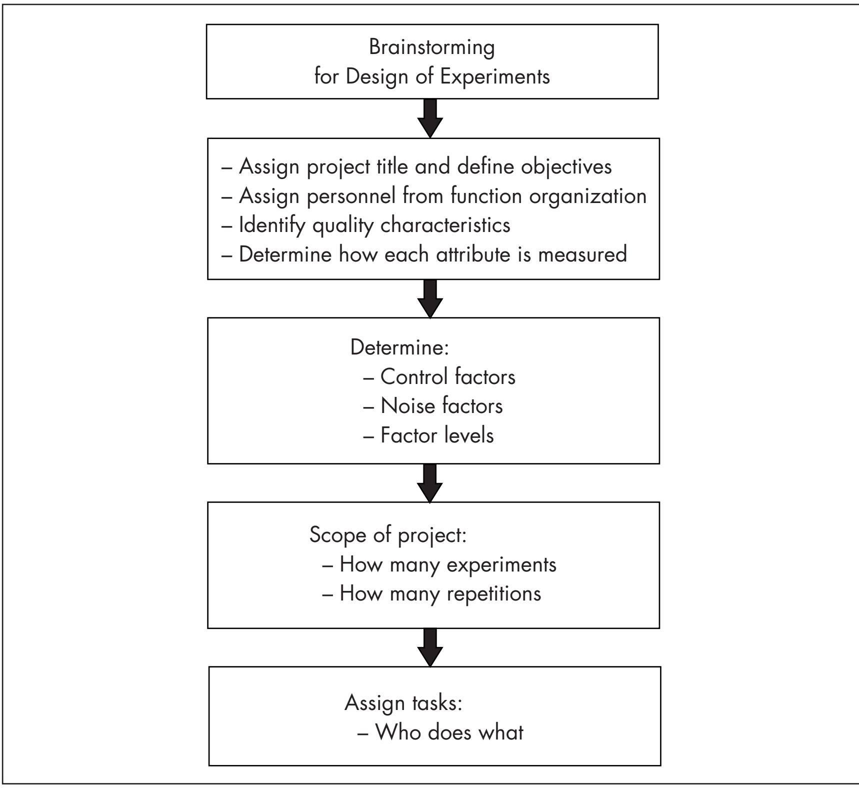


Figure 8-2. Agenda for a brainstorming session

- Brainstorm and qualify factors for study
- Establish levels of study factors
- Select interactions and noise factors for study
- Indicate sample size for trial conditions
- Prescribe sample size and summarize experiment planning discussions

Define the System Under Study

Defining your product or process under study in terms of a system is essential for effective experimental study. The task is to review the product/process flow diagram and define the boundaries of the areas investigated. Depending on the concern or problem that prompts such study, the system may include one or more of the subprocesses that constitutes the performance. For example, in a baking process shown in the flow diagram of Figure 8-3, there are three subprocesses. If the concerns are strictly about mixing and baking, the system may be defined with only the last two subprocesses. Such definition of the system helps the project team more clearly identify input and output of the process under study.

Select Project Title

The team needs to agree on a title for the experimental study. The title is an important identifier for the activities being undertaken. The title needs to be something that relates to the product or process under study. Once the system is defined, the title should describe the scope of the study. For example, if the system only includes mixing subprocess in the cake baking process, the title can be "Cake Ingredients Mixing Process Study." On the other hand, if the system includes both mixing and baking subprocesses, "A Pound Cake Baking Process Parameter Study" may be more appropriate.

Describe Objectives of Experiment

Discuss and agree (majority consensus) on the purpose of launching the study. Describe this in two to five connected sentences or a list of bulleted items. Bear in mind that the project goal may include single or multiple objectives.

No matter how you prefer to define the objectives, it is a good idea to describe in a few brief sentences the reason for the study. The reasons may be to solve a problem, optimize designs, lay out validation tests, or increase response from an advertisement, and so on. No matter the real reason, the activity may be viewed as trying to solve a problem, that is, to fill a void or absence of something. In other words, when you are finished with the experiment, you will obtain something that you do not have now (problem). The project may be stated as a problem you are interested in solving. Here's how the project description can be articulated for a "Plastic Molding" project.

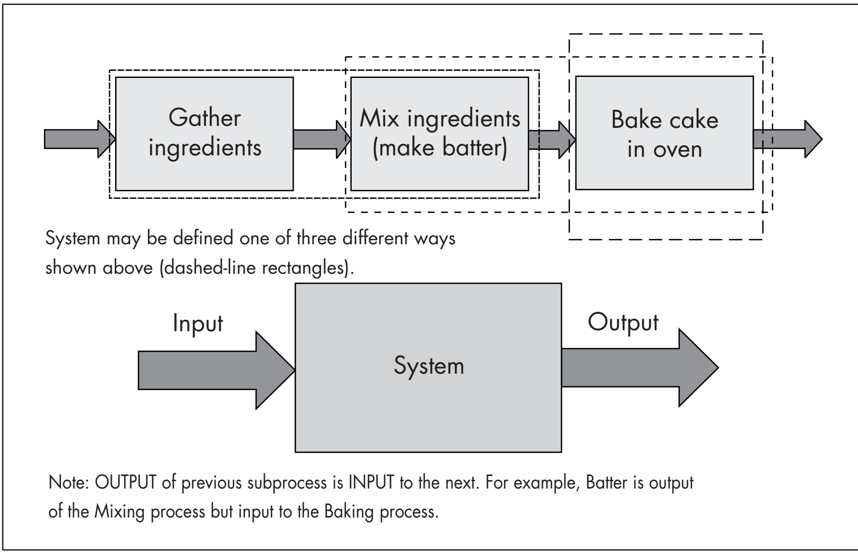


Figure 8-3. System view of process (cake baking)

Example Description

> *"We have been experiencing high rejects and warranty from our plastic molding process. This study is undertaken to determine process parameters that will reduce our scrap rate. The improved process design is also expected to keep our customers satisfied and affect our bottom line."*

The project description may be composed during or after the planning session. Below are a few questions to help describe your projects and define the objectives.

— What are the reasons for performing this project?
— What is it that you want to accomplish with this project?
— What specific objectives/goals you wish to achieve from this project?

If the study involves *baking pound cakes*, the objectives may be considered be to: (a) improve *taste*, (b) increase *moistness*, (c) prolong *shelf life*.

Define Performance (Results) and Units of Measurement

This is an important topic of discussion. The goal is to review all objectives carefully and define the *evaluation criteria* and the units of measurement applicable to each objective. The task can be quite complicated when there are multiple objectives, and it is important to satisfy them all.

Typical discussion can proceed as described here. Suppose that you are to plan a Taguchi experiment to determine the best recipe for the pound cake. The first question is, what are we after? Of course, everyone will agree that they are after the best cake. Obviously, the taste of the cake will be a criterion. But how can the taste be measured? How many will taste it? For a subjective evaluation like taste, evaluation by more than one person is desired. If more than one individual is involved, how can the net result be evaluated? A possible solution is to rate the cakes in terms of a numerical scale, say, on a scale of 0 to 10. Is number 10 for the best taste? This needs to be defined. It can be anything agreed on. But it is important that the matter be discussed, as this rule will dictate the sense of the quality characteristic. If 10 represents the best and 0 the worst, then the quality characteristic becomes "bigger is better." If five people taste the experimental cakes and the ratings vary, whom do we believe? Recognize all of the ratings and record the average of all five evaluations.

Determine Evaluation Criteria and Create OEC Table

When there are multiple criteria involved in evaluating one or more objectives, you may consider combining the evaluations into a single criterion for convenience. The discussions and rationale for this are explained in the following discussions for the cake baking project.

Is taste the only parameter to compare two cake samples? If two cakes have the same taste, can a second parameter distinguish between them? What about the moistness? One individual may prefer a moister over a drier cake. How about shape, appearance, or shelf life? Perhaps moistness is somewhat less important than the taste. The idea of the brainstorming is to raise all of the questions, bring up all of the issues, whether important or not.

How would one measure moistness? By appearance? Perhaps one would decide to take a fixed slice and weigh it and note the weight in ounces. This is an objective measurement. But what is a "standard" slice? One would not necessarily need more than one measurement. Suppose that we agree to consider just these two criteria, that is, taste and moistness. Taste is subjective on a scale of 0 to 10; moistness is measured in ounces. How should one make sense out of the two values? How would we compare a cake rated 7 for taste and 4 ounces for moistness with another rated at 8 for taste and 3.5 ounces for moistness? What index can be devised to distinguish between the two samples? How can the data mix of subjective and objective values be combined and analyzed?

Are these criteria to be given equal weight or unequal weight? How important is moistness compared to taste? Is the moistness one-fourth as important as the taste? Is an assignment such as taste is 80% and moistness is 20% reasonably correct? In other words, if one were to split a dollar among all of the criteria, in accordance with the priority, how would one distribute the 100 cents? When such a question is asked of different groups, the responses are never the same. One group may feel taste is worth 80%, another group may say 60%. As a general rule, when confronted with subjective areas such as these, let consensus prevail. Let everyone participate. Let everyone offer their input. For each of the items, determine the group average.

Suppose that the consensus of the group has been that the weighting of the two quality criteria should be 68% for taste and 32% for moistness. With this knowledge, one can proceed to combine the evaluations and produce a single quantified number called overall evaluation criterion (OEC). The evaluation parameters for the problem can be summarized as in Table 8-1.

Table 8-1. Overall evaluation criteria (OEC)

CRITERIA	VALUE	UNITS	WEIGHTING	MAXIMUM VALUE
Taste	5 (y_1)	None	68 (w_1)	10 (y_{1max})
Moistness	4.5 (y_2)	Ounces	32 (w_2)	6 (y_{2max})

The overall evaluation criterion (OEC) can be defined as:

$$\begin{aligned}\text{OEC} &= \left(y_1/y_{1\max}\right)\times w_1 + \left(y_2/y_{2\max}\right)\times w_2 + \ldots\\ &= (5/10)\times 68/100 + (4.5/6)\times 32/100\\ &= 0.34 + 0.24\\ &= 0.58\end{aligned}$$

where

w_i = weight of ith component
y_i = measurement of ith criterion
$y_{i\max}$ = maximum value of ith criterion

Observe that the evaluation (y) in each case is divided by the maximum value. This is done to get rid of its units (normalization). When multiplied by the weighting, a dimensionless number, the resulting values for each criterion are added to produce a net result in numerical dimensionless terms.

Suppose that an L_8 array is used to describe the eight trial conditions for the experiment. The eight cakes will have to be evaluated following the scheme given above. The OEC calculated above (OEC = 0.58) will represent the result for one trial. There will be seven other results like this. The eight values (OECs) will then form the result column in the orthogonal array (OA). The process will have to be repeated if there were repetitions in each trial condition.

Discussions of factors and levels follow that of quality characteristics. The nature of these issues are based on common sense and some understanding of the problem under investigation. A leader or facilitator will often find it convenient to let their experience determine the flow of the discussions. The remaining portion of the brainstorming session is left up to the reader's imagination.

Brainstorm and Qualify Factors for Study

The discussion and identification of experimental factors should only begin after the objectives and evaluation criteria are defined. To do it any other way will be unwise. The approach to determine factors for study should follow this sequence.

- Solicit ideas and prepare a *long list* of potential factors.
- Scrutinize all ideas and prepare a *qualified list* of factors.
- "Paretoize" the list (from most to least important factors).

Long List

Brainstorm, solicit, and list ideas and suggestions about how to make improvements and what are the possible sources of influence. Realize that, by now, all involved on the team already know what you are after and what are the objectives. The goal here is to capture a quantity of ideas and list them. All ideas gathered do not necessarily make valid factors. However, all suggestions and ideas solicited must be collected without concern for validity. The time for scrutiny and consideration for study will come later.

Below are sample questions that may initiate thoughts about factors:

- What are some of the actions you can take to improve and satisfy performance objectives?
- What are variables (materials/environmental factors/constituents/settings/parameters, etc.) that may influence the outcome of the project?
- If you have done some process studies and have prepared cause-and-effect diagrams (fishbone or Ishikawa diagrams), what are some of the factors that were identified?

If you have a number of people on your project team, this is a good time to ask ideas from each and every person. You do not want to leave "any stone unturned."

If you are alone on the project, or working with a few members on your team, it is a good idea to pause and attempt to collect as many ideas as possible. For the preliminary list of ideas, the longer the list the better chance you have to capture all possible influencing factors.

Qualified List

After you have captured ideas and have a quantity of them listed, you will need to qualify them and identify the valid factors and noise factors by scrutinizing each from the *long list*. Use the following criteria to scrutinize and qualify ideas. The purpose of this exercise is to clean up the list to select only those that are factors (input and controllable). For a factor to be a factor, it must be something that is:

a. An input
b. Controllable at reasonable (or no) cost
c. Adjustable
d. Suspected to have influence on the result
e. Able to be varied independently

The process you should follow is to examine each item in the list and see that it meets one or more of the above criteria. Discard all those that are not factor. Separate those that meet factor criteria but are not controllable or that you do not want to control. Identify them as uncontrollable factors (noise factors) and put them at the bottom of this list.

The result will be a shorter list containing qualified controllable and uncontrollable factors. You should now attempt to gather consensus from the group to place all controllable factors in this list in descending order of importance. A quick way to achieve such group priorities is to ask all on the team to distribute 10 (or 20, depending on the number of factors) pennies to the factors in proportion to personal preferences. Obviously, one will have the option to put some pennies to a few factors and none to others. Add all pennies assigned to each factor and use this number to arrange the factors in descending order. This ordered list of *qualified factors* will help you easily select the factors to include in the study. To decide how many of these factors you can study, follow the logical reasoning described below. For discussion purposes, assume that your *qualified list* comprises 13 factors (listed in descending order) and three noise factors.

Study List

Here you would select factors that you wish to include in the study. Often this list will be shorter than the qualified list.

When you conduct an effective brainstorming with your team, it is very likely that you will identify and qualify a larger number of factors than what is the limit of the size of the experiment. Of course, if money and time are not of concern, you would always want to study all qualified factors with whatever the size of the experiment. Generally, though, the scope will be limited, so you should ask yourself (if you are the team facilitator and/or leader) and others on the team about the scope of the study. Specifically, you would be asking questions such as, how many separate experiments can be done, how many samples can be fabricated, and what test equipment is available. The answers to these questions are very important to help decide the size of the experiment, and consequently, the factors you will be able to include in the study.

Suppose the answer to the number of separate experiments is fewer than 10. In that case, the largest size of the array for your experiment is L-8 or L-9. Understand that at this point you do not know what the levels of the factors need to be or which factors will be included in the study. Your intention will be to include as many factors as possible. So, the strategy here is to select factors first assuming all factors at two levels and then adjust later if some factors need to be at three or four levels. The limit of 10 separate experiments will lead you to select an L-8 array for the experiment, which dictates that you study only seven two-level factors. From the ordered *qualified list*, select the top seven out of 13 factors. For convenience, use symbols/notations of A, B, C, and so on, for these factors. You now have a *study list* of seven factors.

Establish Levels of Study Factors

This discussion should lead to establishing the levels of all factors in the *study list* one at a time. Based on the level requirements, which have not yet been discussed, you may need to modify the scope of the design (array size) after the levels of all factors are determined.

The first issue in determining the level of factors (A, B, C, etc.) is to decide how many levels the factor should have. Generally, all factors should have two levels but may have three or four levels depending on the need. If a factor is a discrete/fixed factor (such as tools, machine, shifts, male/female operator, and so on), it may require more than two levels. Also, if a factor is *known* to have nonlinear behavior, it may be necessary to study it in three or four levels. Otherwise, you should study all factors at two levels when possible, as higher than two levels may cause increased size of the array.

Pick each factor separately to decide its number of levels (two, three, or four) and define the value or description for experimental setup. Use these guidelines:

- The number of levels should be two unless more are required because the factor is discrete or known to be highly nonlinear.
- The values of the factors should be as far away from either side of the current working condition as possible. The levels should be such that the expected results become measurably different under trial-to-trial conditions. The levels should be such that they are practical with which to carry out the tests and that they can easily be released if identified to be part of the optimum condition.

As you complete setting the levels of all factors, should you have factors that require more than two levels (three or four), you will then need to drop factors to make room for this factor (level upgrade) or select for a larger array for the experiment. For example, if you have one of the seven factors at four levels, you will need to drop two of the seven factors to make room for a four-level factor. You will then modify the L-8 array to accommodate this four-level factor (requiring three columns for upgrade) and four remaining two-level factors. However, before you can complete the design process, you will need to consider *interaction* and *noise* factors that might be part of the study and may indeed reshape the experiment.

Select Interactions and Noise Factors for Study

Following selection of factors for the study, interactions and noise factors must be considered. If you choose to study interactions, you may do so by going with a larger array or selecting a few interactions to study in lieu of some factors. Likewise, you need to consider robust design by including noise factors by using an outer array in your experiment.

For interactions, consider only the interactions between two two-level factors (such as $A \times B$, $B \times C$, and so on). Understand that if you have seven two-level factors, there are $7 \times (7 - 1) = 21$ possible interactions. You are now faced with two questions: how many interactions to study, and which ones among all possible ones to study. Generally, you do not have any knowledge to answer these questions. But, if you happen to have the knowledge and/or conviction to decide on some interactions to study, you will have to revise your experiment design. Suppose that you have two interactions that you must study. Because your limit on the size of the experiment is seven columns (in an L-8), you could do so by discarding two factors to make room for the two interactions that have a common factor. If, on the other hand, you want to study all 21 interactions and seven factors, you will require an array that has 28 or more columns. To do so you will need to increase the size of the experiment and go for an L-32 array.

A general recommendation is that you select the biggest array possible and accommodate all factors first. Then if you have spare columns, reserve them to study interactions. This is in line with the philosophy "dig wide and not deep" that Taguchi espouses.

The last item to consider before finalizing the experiment design is to formally incorporate the effects of *noise factors*. The most desirable way to include uncontrollable/noise factors is to go for an outer array design where an orthogonal array is used to formally combine the noise factors. To select robust design conditions, the tests under different recipes of the control factors are repeated by exposing them to the influence of the noise condition so created. The noise factors, of course, are uncontrollable in real life but are assumed to be controllable while conducting the tests in a laboratory environment.

If there are three noise factors (each at two levels) in the experiment, you would use an L-4 array as an outer array. This will require that you run each trial condition (of the control factor) four times by exposing them to four separate noise conditions. Such formal treatment of the noise factors requires more samples and time in carrying out the experiments but is likely to produce more useful information about the system under study.

A general guideline to follow is to go for a robust design approach using an outer array; if this is not possible, carry out multiple sample tests in each trial condition under random noise conditions.

Prescribe Sample Size and Summarize Experiment Planning Discussions

Before you adjourn your planning meeting, you need agree on the number of test samples in each trial condition and share plans for conducting tests, acquiring test facilities, and the data collection procedures with all on the team. If possible, you should also form consensus on the length of time and schedule of completing the study.

Finally, a summary of the information gathered from the planning session will be helpful for the team. This could be a quick review with the group before you adjourn meeting with the team, or prepare it after the meeting and share it with the team members. You will need this summary page when you start using computer software such as [7] to design the experiment. Your planning summary should contain the following information.

Project Title ________________________ Location ______________________

Participants: 1. ___________________ 2. ___________________
3. ___________________ 4. ___________________

Criteria Description	Worst Value	Best Value	QC	Rel. Weighting
1.				
2.				
3.				
etc.				

Your OEC equation (if criteria are combined)
OEC = (   ) x   + (   ) x   + (   ) x   + (   ) x

Example:

FACTORS	Level 1	Level 2	Level 3	Level 4
1.				
2.				
3.				
etc.				

Note: Optionally, list *interactions* and *noise factors* you wish to include in your study. Also, indicate the inner and outer arrays used for the experiment design and how the control factors and noise factors will be assigned to the columns of the arrays. Based on the final design, indicate the test sample size requirements.

EXERCISES

8-1. In an experiment involving the study of an automobile door design, two criteria were used for evaluation purposes. Deflection at a fixed point in the door was measured to indicate the stiffness, and the door closing effort was subjectively recorded on a scale of 0 to 10.

   a. Develop a scheme to define an overall evaluation criterion.
   b. Explain why the overall evaluation may be useful.

8-2. During the brainstorming session for a Taguchi experiment, a large number of factors were initially identified. Discuss the type of information that needs to be considered to determine the number of factors for the experiment, and state how you will proceed to select these factors.

8-3. A group of manufacturing engineers identified the following process parameters for an experimental investigation:

   - Fourteen two-level factors (not all considered important)
   - One interaction between two factors (considered important)
   - Three noise factors at two levels each (considered important)
     If the total number of trial runs (samples) is not to exceed 32, design the experiment and indicate the sizes of the inner and outer arrays.

8-4. Brainstorm and carry out the experiment planning session following the steps discussed in this chapter. The experimental design should be supported by the information provided in the following problem descriptions. Go through the planning process and prepare a summary of experimental data, including title, project objectives, OEC (if applicable), factors for study, and so on.

Problem Description

Engineers and production specialists in a supplier plant wish to optimize the production of foam seats for automobile manufacturers. The improvement project has been undertaken because there have been complaints from the customer about the quality of the delivered parts. The main defects found in the foam parts are: (1) excessive shrinkage, (2) too many voids, (3) inconsistent compression set, and (4) varying tensile strength. There appears to be general agreement that these are the primary objectives; however, there is no consensus as to their relative importance (weighting). Most of the individuals involved are aware that just satisfying one of the criteria may not always satisfy the others. It is believed that a process design that produces parts within the acceptable ranges of all of the objective criteria would be preferable.

Conventional wisdom will dictate that a designed experiment be analyzed separately using the readings for each of the objectives (criteria of evaluations). This way, four separate analyses will have to be performed and optimum design conditions determined. Because each of these optimums is based only on one objective, there is no guarantee that they all will prescribe the same factor levels for the optimum condition. To release the design, however, only one combination of factor levels is desired. Such design must also satisfy all objectives in a manner consistent with the consensus priority established by the project team members.

Combining all of the evaluation criteria into a single index (OEC), which includes the subjective as well as the objective evaluations, and also incorporates the relative weightings of the criteria, may produce the design being sought. Of course, even if the experiment is analyzed using the overall evaluation criteria (OEC), separate analysis may still be performed for individual objectives.

Discussions and investigations into possible causes of the subquality parts revealed many variables (not all are necessarily factors), such as: (a) chemical ratio, (b) mold temperature, (c) lid close time, (d) pour weight, (e) discoloration of surface, (f) humidity, (g) indexing, (h) flow rate, (i) flow pressure, (j) nozzle cleaning time, (k) type of cleaning agent, and so on. Most project team members suspect that there are interactions between the chemical ratio and the pour weight, and between the chemical ratio and the flow rate. Past studies also indicated possible nonlinearity in the influence of the chemical ratio, and thus, four levels of this factor are also desirable for the experiment. But because there have been no scientific studies done in the recent past, any objective evidence of interaction or nonlinearity is not available. Because of the variability from part to part, it is a common practice to study a minimum of three samples for any measurements. The funding and time available for the project is such that only 30 to 35 samples can be molded. (Your plan and answer may vary from others'.)

9 Examples of Taguchi Case Studies

APPLICATION BENCHMARKS

Experiments designed and carried out according to the Taguchi methodology are generally referred to as case studies. Perhaps they are called case studies to indicate that they are well planned experiments and not simply a few tests to investigate the effects of varying one or more factors at a time. The term *case study* may also be used to signify that such planned experiments have been fruitfully carried out, that the results have been analyzed to determine the optimum combination of the factors under study, and that tests to confirm the optimum conditions have been conducted. But what does a case study look like? What are the steps to be followed in completing a case study?

In Chapters 5 and 6, the mechanics of the Taguchi design of experiments and the procedure for the analysis of the experimental data were discussed in detail. Those chapters included several application examples (case studies). The examples in this chapter are representative of practical problems the author has encountered during his associations with various industries and clients.

A typical application of the method will include the following five major steps (see also Figure B-1):

1. A brainstorming session
2. Designing the experiment
3. Conducting the experiment
4. Analyzing the results
5. Running the confirmation test

Brainstorming for Taguchi experiments is described in Chapter 8. Brainstorming is an essential element of a Taguchi case study. When this step is completed, the planning is done. Each of the experimental situations may demand a unique quality objective. What are the attributes of the quality characteristics? In what manner should the results be monitored? How many factors should be included in the study? These and many other pertinent questions are answered in the brainstorming session. Brainstorming was discussed in detail in Chapter 8. In this chapter, the remaining four steps of the Taguchi methodology will be clarified by the following examples. In the solutions of these examples, extensive use is made of computer software [7], which computes results following procedures described in Chapters 5 and 6.

APPLICATION EXAMPLES, INCLUDING DESIGN AND ANALYSIS

Example 9-1
Engine Valve Train Noise Study

An experiment is to be designed to study the influence of six factors, which were identified during brainstorming as influencing the noise emitted by the valve train of a newly developed engine. Each factor is assigned two levels. Brainstorming concluded that interaction effects were much less important than the main effects.

Solution—Example 9-1

Because there are six two-level factors, the smallest array is L_8. Because interactions are insignificant, the six factors can be assigned to the six of the seven columns in any order desired. The factors involved and their levels are shown in Table 9-1(a).

Assume that during the brainstorming session the quality characteristics and the methods of measurement were also determined, in addition to the factors and levels. Based on these criteria, certain key elements of the test plan are described in Table 9-1(a) using the principles of the design of experiments. These are shown under the headings "Note," "Objective," and "Characteristic." For this experiment, the level of the noise was to be measured in terms of some noise index on a scale of 0 to 100. The index was so defined that its smaller value was always desirable.

Table 9-1. Engine valve train noise study (Design)—Example 9-1

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Valve guide clearance	Low	High		
2	Upper guide length	Smaller	Larger		
3	Valve geometry	Type 1	Type 2		
4	Seat concentricity	Quality 1	Quality 2		
5	Lower guide length	Location 1	Location 2		
6	Valve face runout	Runout type 1	Runout type 2		
7	(Unused)				

Note: Six variables all at two levels studied.
Objective: Determine design configuration for least noise.
Characteristic: Smaller is better (measured in terms of noise index).

(b) L_8 Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4	5	6	7
Trial 1	1	1	1	1	1	1	0
Trial 2	1	1	1	2	2	2	0
Trial 3	1	2	2	1	1	2	0
Trial 4	1	2	2	2	2	1	0
Trial 5	2	1	2	1	2	1	0
Trial 6	2	1	2	2	1	2	0
Trial 7	2	2	1	1	2	2	0
Trial 8	2	2	1	2	1	1	0

The L_8 array is shown in Table 9-1(b). Note that only six columns define the test condition, with the zeros in the unused column (column 7) showing that no condition is implied. The two-level array, L_8, describes eight trial conditions. The design may be created manually, but a computer program will perform such computations in a matter of seconds and without mathematical errors.

The results of the eight trial conditions, with one run per trial condition, are shown in Table 9-2(a). Examples in this chapter utilized computer software [7], which displays up to six repetitions and their averages. These observed results are used to compute the main effects of the individual factors [Table 9-2(b)]. Because

Table 9-2. Original data and their averages (Results and analysis)—Example 9-1

(a) Original Observations and Their Averages
Quality Characteristic: Smaller is Better
Results: Up to Six Repetitions Shown

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	45.00						45.00
2	34.00						34.00
3	56.00						56.00
4	45.00						45.00
5	46.00						46.00
6	34.00						34.00
7	39.00						39.00
8	43.00						43.00

(b) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Valve guide clearance	45.00	40.50	–4.50	00.00	00.00
2	Upper guide length	39.75	45.75	6.00	00.00	00.00
3	Valve geometry	40.25	45.25	5.00	00.00	00.00
4	Seat concentricity	46.50	39.00	–7.50	00.00	00.00
5	Lower valve length	44.50	41.00	–3.50	00.00	00.00
6	Valve face runout	44.75	40.75	–4.00	00.00	00.00

(c) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
1	Valve guide clear.	(1)	(40.50)	Pooled		
2	Upper guide length	1	72.00	72.00	2.011	9.96
3	Valve geometry	(1)	(50.00)	Pooled		
4	Seat concentricity	1	112.50	112.50	3.142	21.10
5	Lower guide length	(1)	(24.50)	Pooled		
6	Valve face runout	(1)	(32.00)	Pooled		
All other/error		5	179.00	35.80		68.94
Total:		7	363.50			100.00

Note: Insignificant factorial effects are pooled as shown ( ).

the factors have only two levels, the main effects are shown under the two columns marked Level 1 and Level 2. The third column labeled $(L_2 - L_1)$ contains the difference between the main effects at Level 1 and Level 2. A minus sign (in the difference column) indicates a decrease in noise as the factor changes from Level 1 to Level 2. A positive value, on the other hand, indicates an increase in noise. A quick inspection of the difference column permits selection of the optimum combination, for example, the "smaller is better" characteristic. A negative sign in the column $(L_2 - L_1)$ indicates Level 2 of the factor is desirable, while a positive value indicates Level 1 is the choice. This quick inspection is a sufficient test only when two levels are involved and when all factors are considered significant.

If the desired characteristic is "bigger is better," then the level selection criteria will be the reverse of the scheme given above; positive values indicate Level 2, and all negative values will indicate the choice of Level 1 for the optimum condition. In this example with all factors, the optimum condition for "smaller is better" is levels 2, 1, 1, 2, 2, and 2 for factors in columns 1 through 6, respectively. The sign $(\pm)$ directs the selection of levels, while the magnitude suggests the strength of the influence of the factor. The quantitive measure of the influence of individual factors is obtained from ANOVA [Table 9-2(c)].

ANOVA follows procedures outlined in Chapter 6. No new data or decisions on the part of the experimenter are required. This is an ideal situation for standard computer routines. The results of ANOVA are shown in Table 9-2(c). A review of the percent column shows that Upper Guide (9.96%) and Seat Concentricity (21.10%) are significant. The other insignificant factors are pooled (combined) with the error term. Based on information from the ANOVA Table 9-2(c), the mean performance at optimum condition and the confidence interval are calculated as shown in Tables 9-3(a) and 9-3(b), respectively.

The last step in the analysis is to estimate the performance at the optimum condition. Normally only the significant factors are used for this estimate. An examination of main effects indicates which levels will be included in the optimum condition. In addition, ANOVA

Table 9-3. Engine valve train noise study (Optimum and confidence interval)—Example 9-1

(a) Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Upper guide length	Smaller	1	–3.0000
Seat concentricity	Quality	2	–3.7500
Contribution from all factors (total)			–6.75
Current grand average of performance			42.75
Expected result at optimum condition			36.00

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

(b) Confidence Interval

Computing F function for 1 and 5 at 90% confidence level.
Confidence Interval (C.I.) is expressed as:

$$\text{C.I.} = \sqrt{\frac{F(1, n_2) V_e}{N_e}}$$

where $F(n_1, n_2)$ = computed value of F with $n_1 = 1$, n_2 = error DOF at a desired confidence level

V_e = error variance
N_e = effective number of replications
Based on: $F = 3.2999999$, $n_1 = 1$, $n_2 = 5$, $V_e = 35.8$, $N_e = 2.6667$ [from 8/(1+2)]

The confidence interval C.I. = ±6.656011, which is the variation of the estimated result at the optimum condition; that is, the mean of the result, m, lies between (m + C.I.) and (m – C.I.) at 89.93% confidence level.

indicates [by the percentage column in Table 9-2(c)] the relative influence of each factor. Thus, all of the necessary information for determination of the optimum condition and the expected value of the response at this condition is available. No new information is necessary to calculate the performance at the optimum condition.

Example 9-2
Study of Crankshaft Surface Finishing Process

An engine was found to have an unusually high rate of crankshaft bearing failures. Engineers identified the crankshaft surface finish as the root cause. A brainstorming session with the engineers and the technicians involved in design and manufacturing activities resulted in the selection of six factors that were considered to have a major influence on the quality of the surface finish. The Taguchi approach of experimental design was considered an effective way to optimize the process.

The brainstorming also identified two levels for each factor and a likely interaction between two of the factors. The group decided that the quality characteristic of the surface finish should be measured in terms of durability (life) under simulated laboratory tests.

Solution—Example 9-2

With six factors and one interaction involved in this study, an L_8 orthogonal array (OA) is suitable for the experimental design. The first step is to decide where to assign the interacting factors and which column to reserve for their interaction. The table of interaction (Table A-6) for two-level orthogonal arrays shows that columns 1, 2, and 3 form an interacting group. The two interacting factors are therefore assigned to columns 1 and 2. Column 3 is kept aside for their interaction. The remaining four factors are then assigned to any of the four remaining columns. The completed design, with descriptions of factors, their levels, and the orthogonal array, are shown in Tables 9-4(a) and 9-4(b). Eight crankshafts were fabricated to the specifications described by the eight trial conditions. Each sample was tested for durability (life). Because longer life was desirable, the quality characteristic applicable in this case was "bigger is better."

The observed durability, the main effects, and the unpooled ANOVA are shown in Table 9-5. The study of the main effects indicates some interaction between the factors. This is shown by the magnitude 1.25 in the column labeled $(L_2 - L_1)$ in Table 9-5(b). This value is of the same order of magnitude as the values 3.25, –6.25, 4.25, and so on. But is the interaction significant? The answer to this question can be obtained from the percentage column of the ANOVA table [Table 9-5(c)]. The interaction under column 3 is only 0.74%. Contributions below 5% are generally not considered significant. The interaction and the factor in column 5, which has 1.46 in the percentage column, are pooled. The pooled ANOVA is

Table 9-4. Study of crankshaft surface finishing process (Design)—Example 9-2

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Roundness (lobing)	700	1600		
2	Lay direction (cross)	Least	Most		
3	Interaction	N/A			
4	T_p (process index)	65%	90%		
5	Taper	.00025	.0005		
6	Waviness	Lower limit	Upper limit		
7	Shape factor	.0003	.0003		

Note: Interaction between roundness and lay direction.
Objective: Determine best grinding parameters.
Characteristic: Bigger is better (bearing durability life).

(b) L_8 Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4	5	6	7
Trial 1	1	1	1	1	1	1	1
Trial 2	1	1	1	2	2	2	2
Trial 3	1	2	2	1	1	2	2
Trial 4	1	2	2	2	2	1	1
Trial 5	2	1	2	1	2	1	2
Trial 6	2	1	2	2	1	2	1
Trial 7	2	2	1	1	2	2	1
Trial 8	2	2	1	2	1	1	2

shown in Table 9-6(a). Observe that upon pooling the percentage values the significant factors are adjusted slightly.

In estimating performance at the optimum, only significant factors are used. As in Table 9-6(b), the expected improvement in performance is 14.875 over the current average of performance (44.125). Because the interaction [Table 9-6(b)] has little significance, it is not considered in the selection of levels for the optimum condition.

Table 9-5. Crankshaft surface finishing process (Main effects and ANOVA)—Example 9-2

(a) Original Observations and Their Averages
Quality Characteristic: Bigger is Better
Results: Up to Six Repetitions Shown

TRIAL \ REPETITION	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	34.00						34.00
2	56.00						56.00
3	45.00						45.00
4	35.00						35.00
5	46.00						46.00
6	53.00						53.00
7	43.00						43.00
8	41.00						41.00

(b) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Roundness (lobing)	42.50	45.75	3.25	00.00	00.00
2	Lay direction	47.25	41.00	–6.25	00.00	00.00
3	Interaction	43.55	44.75	1.25	00.00	00.00
4	T_p (process index)	42.00	46.25	4.25	00.00	00.00
5	Taper	43.25	45.00	1.75	00.00	00.00
6	Waviness	39.00	49.25	10.25	00.00	00.00
7	Shape factor	41.25	47.00	5.75	00.00	00.00

(c) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
1	Roundness	1	21.13	21.13		5.02
2	Lay direction	1	78.13	78.13		18.56
3	Interaction	1	3.13	3.13		0.74
4	T_p (process index)	1	36.13	36.13		8.58
5	Taper	1	6.13	6.13		1.46
6	Waviness	1	210.13	210.13		49.93
7	Shape factor	1	66.13	66.13		15.71
All other/error		0				
Total:		7	420.91			100.00

Table 9-6. Crankshaft surface finishing process (Pooled ANOVA and optimum)—Example 9-2

(a) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
1	Roundness	1	21.13	21.13	4.57	3.92
2	Lay direction	1	78.13	78.13	16.89	17.46
3	Interaction	(1)	(3.13)	Pooled		
4	T_p (process index)	1	36.13	36.13	7.81	7.48
5	Taper	(1)	(6.13)	Pooled		
6	Waviness	1	210.13	210.13	45.43	48.83
7	Shape factor	1	66.13	66.13	14.30	14.61
All other/error		2	9.25	4.63		7.69
Total:		7	420.91			100.00

Note: Insignificant factorial effects are pooled as shown (   ).

(b) Estimate of Optimum Condition of Design/Process:
For Bigger is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Roundness (lobing)	1600	2	1.6250
Lay direction (cross)	Least	1	3.1250
T_p (process index)	90%	2	2.1250
Waviness	Upper limit	2	5.1250
Shape factor	.0003	2	2.8750
Contribution from all factors (total)			14.875
Current grand average of performance			44.125
Expected result at optimum condition			59.00

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

Example 9-3
Automobile Generator Noise Study

Engineers identified one four-level factor and four two-level factors for experimental investigation to reduce the operating noise of a newly released generator. Taguchi methodologies were followed to lay out the experiments and analyze the test results.

Solution—Example 9-3

The factors in this example present a mixed-level situation. Although experiment design is simplified if all factors have the same level, it is not always possible to compromise the factor level. For instance, if a factor influence is believed to be nonlinear, it should be assigned three or more levels. The factor and its influence are assumed to be continuous functions. If, however, the factor assumes discrete levels such as design type 1, design type 2, and so on, then the influence is a discrete function, and each discrete step (level) must be incorporated in the design. The four-level factor in the example is discrete. Because the four-level factor has 3 DOF, and four two-level factors each have 1 DOF, the total DOF for the experiment is 7. An L_8 with seven two-level columns and 7 DOF was selected for the design. The first step provides for the four-level factor. Columns 1, 2, and 3 of L_8 are used to prepare a four-level column. This new four-level column now replaces column 1 and is assigned to the four-level factor. As columns 2 and 3 were used to prepare column 1 as a four-level column, they cannot be used for any other factor. Thus, the four two-level factors are assigned to the remaining columns 4, 5, 6, and 7. The design and the modified OA are shown in Tables 9-7(a) and 9-7(b).

One run at each trial condition was tested in the laboratory, and the performance was measured in terms of a noise index. The index ranged between 0 (low noise) and 100 (loud noise). The lower value of this index was desirable. The data and calculated main effects are shown in Tables 9-8(a) and 9-8(b), respectively. Note that the four-level factor in column 1 (Casement Structure) has its main effects at the four levels. This factor has 3 DOF as noted in the ANOVA table [Table 9-9(a)] under the column marked DOF.

The ANOVA table clearly shows that the factor in column 6 (Contact Brushes) has the smallest sum of squares and hence the least influence. This factor is pooled and the new ANOVA is in Table 9-9(a). Using the significant contributors, the estimated performance at the optimum condition was calculated as 49.375. In this case, the optimum condition is trial 1 (Levels 1 1 1 1 1). The result for trial 1 was 50 [Table 9-8(a)]. The difference between the trial result and the

Table 9-7. Automobile generator noise (Design)—Example 9-3

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Casement structure	Present	Textured	Ribbed	New
2	(Unused)				
3	(Unused)				
4	Air gap	Present	Increase		
5	Impregnation	Present type	Harder type		
6	Contact brush	Type 1	Type 2		
7	Stator structure	Present design	Epoxy coated		

Note: One four-level and four two-level factors studied.
Objective: Determine generator design parameters for least noise.
Characteristic: Smaller is better (measured in predefined index).

(b) L_8 Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4	5	6	7
Trial 1	1	0	0	1	1	1	1
Trial 2	1	0	0	2	2	2	2
Trial 3	2	0	0	1	1	2	2
Trial 4	2	0	0	2	2	1	1
Trial 5	3	0	0	1	2	1	2
Trial 6	3	0	0	2	1	2	1
Trial 7	4	0	0	1	2	2	1
Trial 8	4	0	0	2	1	1	2

estimated optimum performance (49.375) resulted from the dropping of the minor effect of the contact brush factor from the estimate.

Example 9-4
Engine Idle Stability Study

An engine development engineer identified three adjustment parameters controlling the idle performance of an engine. Each of the factors is to be studied at three levels to determine the best setting for the engine. A Taguchi experiment design is to be utilized.

Table 9-8. Automobile generator noise (Main effects)—Example 9-3

(a) Original Observations and Their Averages
Quality Characteristic: Smaller is Better
Results: Up to Six Repetitions Shown

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	50.00						50.00
2	62.00						62.00
3	70.00						70.00
4	75.00						75.00
5	68.00						68.00
6	65.00						65.00
7	65.00						65.00
8	74.00						74.00

(b) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Casement structure	56.00	72.50	16.50	66.50	69.50
4	Air gap	63.25	69.00	5.75	00.00	00.00
5	Impregnation	64.75	67.50	2.75	00.00	00.00
6	Contact brush	66.75	65.50	–1.25	00.00	00.00
7	Stator structure	63.75	68.50	4.75	00.00	00.00

Solution—Example 9-4

The smallest three-level OA, L_9, has four three-level columns. With three three-level factors in this study, the L_9 is appropriate for the design. The factors are placed in the first three columns, leaving the fourth column unused. The factors, their levels, and the modified OA are shown in Tables 9-10(a) and 9-10(b).

The performance of the engine tested under various trial conditions was measured in terms of the deviation of the speed from a nominal idle speed. A smaller deviation represented a more stable condition. Three separate observations were recorded for each trial condition, as shown in Table 9-11(a). The signal-to-noise (S/N) ratio was used for the analysis of the results. The main effects, optimum condition, and ANOVA table are shown in Tables 9-11 and 9-12. Based on the error DOF and variance, the confidence

Table 9-9. Automobile generator noise (Pooled ANOVA and optimum)—Example 9-3

(a) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	Casement structure	3	309.38	103.13	33.00	70.49
4	Air gap	1	66.13	66.13	21.16	15.07
5	Impregnation	1	15.13	15.13	4.84	3.45
6	Contact brushes	(1)	(3.13)	Pooled		
7	Stator structure	1	45.13	45.13	14.44	9.57
All other/error		1	3.13	3.12		4.98
Total:		7	438.88			100.00

Note: Insignificant factorial effects are pooled as shown ( ).

(b) Estimate of Optimum Condition of Design/Process:
For Bigger is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Casement structure	Present design	1	–10.1250
Air gap	Present gap	1	–2.8750
Impregnation	Present type	1	–1.3750
Stator structure	Present design	1	–2.3750
Contribution from all factors (total)			–16.750
Current grand average of performance			66.125
Expected result at optimum condition			49.375

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

interval of the estimated performance at optimum is also computed as shown in Table 9-12(b). The confidence interval (C.I.) value of $\pm.3341$ will mean that the estimated optimum performance (S/N ratio) will be $-25.878 \pm .3341$ at 90% confidence level (89.77% as a result of numerical solution by computer software [7]).

Example 9-5
Instrument Panel Structure Design Optimization

A group of analytical engineers undertaking the design of an instrument panel structure are to study the influence of five critical

Table 9-10. Engine idle stability study (Design)—Example 9-4

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Indexing	–5 deg	0 deg	+5 deg	
2	Overlap area	0%	30%	60%	
3	Spark advance	25 deg	30 deg	35 deg	
4	(Unused)				

Note: Three three-level factors studied.
Objective: Determine best engine setting.
Characteristic: Smaller is better (speed deviation).

(b) L_9 Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4
Trial 1	1	1	1	0
Trial 2	1	2	2	0
Trial 3	1	3	3	0
Trial 4	2	1	3	0
Trial 5	2	2	1	0
Trial 6	2	3	2	0
Trial 7	3	1	2	0
Trial 8	3	2	3	0
Trial 9	3	3	1	0

structural modifications on the system. A finite element model of the total structure was available for a static stiffness analysis. The objective is to determine the best combination of design alternatives. To reduce the number of computer runs, a Taguchi experiment design was selected to determine the number and conditions of the computer runs necessary if each factor is to be studied at two levels. Two interactions are believed to be important.

Solution—Example 9-5

This investigation is an analytical simulation rather than a hardware experiment. The factors and levels shown in Table 9-13(a) are used in an L_9 OA to set up the simulation. Only one

Table 9-11. Engine idle stability study (Main effects and ANOVA)—Example 9-4

(a) Original Observations and Their S/N Ratios
Quality Characteristic: Smaller is Better
Results: Up to Six Repetitions Shown

TRIAL \ REPETITION	R_1	R_2	R_3	R_4	R_5	R_6	S/N
1	20.00	25.00	26.00				–27.54
2	34.00	36.00	26.00				–30.19
3	45.00	34.00	26.00				–31.10
4	13.00	23.00	22.00				–25.96
5	36.00	45.00	35.00				–31.81
6	23.00	25.00	34.00				–28.87
7	35.00	45.00	53.00				–33.06
8	56.00	46.00	75.00				–35.60
9	35.00	46.00	53.00				–33.12

(b) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Indexing	–29.61	–28.88	0.73	–33.93	00.00
2	Overlap	–28.85	–32.53	–3.69	–31.03	00.00
3	Spark advance	–30.67	–29.76	0.91	–31.99	00.00

(c) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	Indexing	2	44.636	22.318	932.80	61.26
2	Overlap	2	20.541	10.271	429.27	28.15
3	Spark advance	2	7.565	3.783	158.10	10.33
All other/error		2	0.05			0.26
Total:		8	72.79			100.00

run per trial condition is necessary because the computer results should not change with repetition. The observation (stiffness values), main effects, and optimum condition are shown in Tables 9-13(b), 9-14(a), and 9-14(b). Note that the estimate of optimum performance (25.25) is lower than the result (29.9) of Trial #7 as it represents a conservative value that uses only three significant factors.

Table 9-12. Engine idle stability study (Optimum and confidence interval)—Example 9-4

(a) Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Indexing	0 deg	2	1.9256
Overlap	0%	1	1.9522
Spark direction	30 deg	2	1.0489
Contribution from all factors (total)			4.92667
Current grand average of performance			–30.80556
Expected result at optimum condition			–25.87889

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

(b) Confidence Interval

Computing *F* function for 1 and 2 at 90% confidence level.
Confidence Interval (C.I.) is expressed as:

$$\text{C.I.} = \sqrt{\frac{F(1, n_2)V_e}{N_e}}$$

where $F(n_1, n_2)$ = computed value of F with $n_1 = 1$, n_2 = error DOF at a desired confidence level

V_e = error variance
N_e = effective number of replications
Based on: $F = 5.999996$, $n_1 = 1$, $n_2 = 2$, $V_e = 2.392578\text{E}$,
$N_e = 1.285714$ [from 9/(1+6)]

The confidence interval C.I. = ±0.3341461, which is the variation of the estimated result at the optimum condition; that is, the mean of the result, *m*, lies between (*m* + C.I.) and (*m* – C.I.) at 89.77% confidence level.

Example 9-6
Study Leading to Selection of Worst-Case Barrier Test Vehicle

To assure that the design of a new vehicle complies with all of the applicable Federal Motor Vehicle Safety Standards (FMVSS) requirements, engineers involved in the crashworthiness development of a new vehicle design want to determine the worst combination of vehicle body style and options. This vehicle is to be used as the test specimen for laboratory validation tests instead of

Table 9-13. Instrument panel structure optimization (Design and data)—Example 9-5

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Front dash beam	Solid	Hollow		
2	Understructure	With	Without		
3	Interaction 1×2	N/A			
4	Forward panel	Current design	New design		
5	Interaction 1×4	N/A			
6	Plenum structure	Steel	Plastic		
7	Surface structure	Baseline	New		

Note: Interactions 1×2 and 1×4 studied.
Objective: Determine structural parameters for maximum strength.
Characteristic: Bigger is better (measured in terms of stiffness).

(b) Original Observations and Their Averages
Quality Characteristic: Bigger is Better
Results: Up to Six Repetitions Shown

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	13.50						13.50
2	14.00						14.00
3	14.30						14.30
4	13.10						13.10
5	22.00						22.00
6	18.00						18.00
7	29.90						29.90
8	16.00						16.00

subjecting several prototype vehicles with all available options and body styles to tests under all compliance conditions. Four two-level factors and one four-level factor were considered to have major influence on the performance. A Taguchi experimental design approach was followed.

Solution—Example 5-6

The design involved modifying a two-level column of an L_8 into a four-level one. The process is similar to that described in

Table 9-14. Instrument panel structure optimization (Main effects and optimum)—Example 9-5

(a) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Front dash beam	–29.61	–28.88	0.73	–33.93	00.00
2	Understructure	–28.85	–32.53	–3.69	–31.03	00.00
3	Interaction 1×2					
4	Forward panel					
5	Interaction 1×4					
6	Plenum structure					
7	Surface structure	–30.67	–29.76	0.91	–31.99	00.00

(b) Estimate of Optimum Condition of Design/Process:
For Bigger is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Front dash beam	Hollow	2	3.8750
Forward panel	Current design	1	2.3250
Plenum structure	Plastic	2	1.4500
Contribution from all factors (total)			7.64999
Current grand average of performance			17.64000
Expected result at optimum condition			25.25000

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

Example 7-3. The factors, their levels, and the modified OA are shown in Tables 9-15(a) and 9-15(b). The description of the trial conditions derived from the designed experiment served as the specifications for the test vehicle. For barrier tests, the specimens are prototype vehicles built either on the production line or are handmade, one-of-a-kind test vehicles. In either case, the cost for preparing the test vehicles could easily run in the hundreds of thousands of dollars. Proper specification, in a timely manner, is crucial to the cost effectiveness of the total vehicle development program. For the purpose of the tests, eight vehicles were built on the production line following the specifications that correspond to the eight trial conditions. These vehicles were barrier tested and the results recorded in terms of a predefined occupant injury

Table 9-15. Selection of worst-case barrier vehicle (Design)—Example 9-6

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Test type	0 deg F	30 deg R	30 deg L	NCAP
2	(Unused)				
3	(Unused)				
4	Type of vehicle	Style 1	Style 2		
5	Powertrain	Light duty	Heavy duty		
6	Roof structure	Hard top	Sunroof		
7	Seat structure	Standard	Reinforced		

Note: One four-level and four two-level factors studied.
Objective: Determine the worst vehicle/option combination.
Characteristic: Smaller is better (one or more injury criteria).

(b) L_8 Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4	5	6	7
Trial 1	1	0	0	1	1	1	1
Trial 2	1	0	0	2	2	2	2
Trial 3	2	0	0	1	1	2	2
Trial 4	2	0	0	2	2	1	1
Trial 5	3	0	0	1	2	1	2
Trial 6	3	0	0	2	1	2	1
Trial 7	4	0	0	1	2	2	1
Trial 8	4	0	0	2	1	1	2

index. The results and the analyses are shown in Tables 9-16 and 9-17. By using eight test vehicles, the engineers were able to learn the worst vehicle configuration. This information was then used to adhere to several of the compliance regulations.

Example 9-7
Airbag Design Study

Engineers involved in the development of an impact-sensitive inflatable airbag for automobiles identified nine four-level

Table 9-16. Selection of worst-case barrier vehicle (Main effects and ANOVA)—Example 9-6

(a) Original Observations and Their Averages
Quality Characteristic: Smaller is Better
Results: Up to Six Repetitions Shown

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	AVERAGE
1	45.00						45.00
2	65.00						65.00
3	38.00						38.00
4	48.00						48.00
5	59.00						59.00
6	32.00						32.00
7	36.00						36.00
8	38.00						38.00

(b) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Test type	55.00	43.00	–12.00	45.50	37.00
4	Type of vehicle	44.50	45.75	1.25	00.00	00.00
5	Powertrain	38.25	52.00	–13.75	00.00	00.00
6	Roof structure	47.50	42.75	–4.75	00.00	00.00
7	Seat structure	40.25	50.00	9.75	00.00	00.00

(c) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	*F*	PERCENT
1	Test type	3	336.38	336.38	4.648	27.71
4	Type of vehicle	(1)	(3.13)	Pooled		
5	Powertrain	1	378.13	378.13	15.674	37.15
6	Roof structure	(1)	(45.13)	Pooled		
7	Seat structure	1	190.13	190.13	7.881	17.42
All other/error		2	48.25	24.13		17.72
Total:		7	952.88			100.00

Note: Insignificant factorial effects are pooled as shown ( ).

factors as the major influences on performance. With this information, Taguchi experimental design was used to determine the optimum design.

Table 9-17. Selection of worst-case barrier vehicle (Optimum and confidence interval)—Example 9-6

(a) Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Test type	NCAP	4	–8.1250
Powertrain	Light duty	1	–6.8750
Seat structure	Standard	1	–4.8750
Contribution from all factors (total)			–19.8750
Current grand average of performance			45.1250
Expected result at optimum condition			25.2500

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

(b) Confidence Interval

Computing F function for 1 and 2 at 90% confidence level.
Confidence Interval (C.I.) is expressed as:

$$\text{C.I.} = \sqrt{\frac{F(1, n_2)V_e}{N_e}}$$

where $F(n_1, n_2)$ = computed value of F with $n_1 = 1$, n_2 = error DOF at a desired confidence level

V_e = error variance
N_e = effective number of replications

Based on: $F = 5.999996$, $n_1 = 1$, $n_2 = 2$, $V_e = 24.125$,
$N_e = 1.333333$ [from 8/(1+5)]

The confidence interval C.I. = ±10.4

Solution—Example 9-7

Because the experiment involves nine four-level factors, an L_{32} with nine four-level columns and one two-level column was selected for the design. Because there is no two-level factor in this design, the two-level column (column 1) of the OA shown in Table 9-18(b) is set to zero. The factors, their levels, and the analyses are shown in Tables 9-18(a) through 9-20(b). The study was done using a theoretical simulation of the system. The trial conditions were used to set up the input conditions for the computer runs. The results of the computer runs at each of the trial conditions were recorded on a scale of 1 to 10 and are as shown with the OA (right-most column) in Table 9-18(b). The main effects and ANOVA are presented in Tables 9-19(a) and 9-19(b). The optimum vehicle option combination and confidence level of the design appear in Table 9-20(a) and 9-20(b).

Example 9-8
Transmission Control Cable Adjustment Parameters

A Taguchi experiment was conducted to determine the best parameters for the design of a transmission control cable. The engineers identified one four-level factor and five two-level factors as well as three interactions among three of the five two-level factors.

Solution—Example 9-8

This experiment required both level of modification and interaction study. The total DOF for the experiment was 11 [$(4-1) + 5 \times (2-1) + 3 \times (1 \times 1)$]. L_{12} has 11 DOF. However, it requires a special OA that cannot be used for interaction studies. L_{16} is selected for the design. The factors and their levels are described in Table 9-21(a). For a four-level column and for the three interactions, four groups of natural interaction columns are first selected. The sets selected are 1, 2, 3; 7, 8, 15; 11, 4, 15; and 12, 4, 8. Columns 1, 2, and 3 are used to upgrade column 1 into a four-level column. The other three sets are reserved for the interactions among the three factors assigned to columns 4, 8, and 15 such that interaction 4×15 is shown in column 11, interaction 4×8 in column 12, and interaction 8×15 in column 7. The factor with four levels is assigned to column 1, which is now a four-level column. The remaining two two-level columns are assigned to columns 5 and 6. Columns 9, 10, 13, and 14 remain unused. The modified L_{16} and the factors assigned to the appropriate columns are shown in Table 9-21(a).

Example 9-9
Front Structure Crush Characteristics

The Taguchi design of experiments methodology was used to optimize the design of the basic load-carrying members of an automobile front structure. The development engineers were interested in determining the best combination of designs with three

Table 9-18. Airbag design study (Design)—Example 9-7

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	(Unused)				
2	Steering column rotation	Rot 1	Rot 2	Rot 3	Rot 4
3	Steering column crush stiffness	300	350	400	500
4	Knee bolster stiffness	S1	S2	S3	S4
5	Knee bolster location	100 mm	115 mm	150 mm	175 mm
6	Inflation rate	Rate 1	Rate 2	Rate 3	Rate 4
7	Development time	14 ms	18 ms	24 ms	32 ms
8	Vent size	650 mm	970 mm	1300 mm	1625 mm
9	Bag size (E-7 mm)	5.0	5.8	6.5	7.2
10	Maximum bag pressure	P1	P2	P3	P4

(b) L_{32} Orthogonal Array Used for Experiment

COLUMN / TRIAL	1	2	3	4	5	6	7	8	9	10	R1
Trial 1	0	1	1	1	1	1	1	1	1	1	8.00
Trial 2	0	1	2	2	2	2	2	2	2	2	5.50
Trial 3	0	1	3	3	3	3	3	3	3	3	5.00
Trial 4	0	1	4	4	4	4	4	4	4	4	7.00
Trial 5	0	2	1	1	2	2	3	3	4	4	8.00
Trial 6	0	2	2	2	1	1	4	4	3	3	4.00
Trial 7	0	2	3	3	4	4	1	1	2	2	5.00
Trial 8	0	2	4	4	3	3	2	2	1	1	7.00
Trial 9	0	3	1	2	3	4	1	2	3	4	8.00
Trial 10	0	3	2	1	4	3	2	1	4	3	3.00
Trial 11	0	3	3	4	1	2	3	4	1	2	5.00
Trial 12	0	3	4	3	2	1	4	3	2	1	4.00
Trial 13	0	4	1	2	4	3	3	4	2	1	6.00
Trial 14	0	4	2	1	3	4	4	3	1	2	4.00
Trial 15	0	4	3	4	2	1	1	2	4	3	7.00
Trial 16	0	4	4	3	1	2	2	1	3	4	5.00
Trial 17	0	1	1	4	1	4	2	3	2	4	3.00
Trial 18	0	1	2	3	2	3	1	4	1	4	5.00
Trial 19	0	1	3	2	3	2	4	1	4	1	7.00
Trial 20	0	1	4	1	4	1	3	2	3	2	9.00
Trial 21	0	2	1	4	2	3	4	1	3	2	5.00
Trial 22	0	2	2	3	1	4	3	2	4	1	6.00
Trial 23	0	2	3	2	4	1	2	3	1	4	7.00
Trial 24	0	2	4	1	3	2	1	4	2	3	7.00

(continued)

Table 9-18 (continued)

COLUMN / TRIAL	1	2	3	4	5	6	7	8	9	10	R1
Trial 25	0	3	1	3	3	1	2	4	4	2	6.00
Trial 26	0	3	2	4	4	2	1	3	3	1	5.00
Trial 27	0	3	3	1	1	3	4	2	2	4	4.00
Trial 28	0	3	4	2	2	4	3	1	1	3	5.00
Trial 29	0	4	1	3	4	2	4	2	1	3	6.00
Trial 30	0	4	2	4	3	1	3	1	2	4	7.00
Trial 31	0	4	3	1	2	4	2	4	3	1	8.00
Trial 32	0	4	4	2	1	3	1	3	4	2	4.50

Table 9-19. Airbag design study (Main effects and ANOVA)—Example 9-7

(a) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
2	Steering column rotation	6.19	6.13	–0.07	5.00	5.94
3	Steering column crush stiffness	6.25	4.94	–1.32	6.00	6.06
4	Knee bolster stiffness	6.38	5.88	–0.50	5.25	5.75
5	Knee bolster location	4.94	5.94	1.00	6.38	6.00
6	Inflation rate	6.50	6.06	–0.44	4.94	5.75
7	Development time	6.19	5.56	–0.63	6.38	5.13
8	Vent size	5.63	6.56	0.93	5.06	6.00
9	Bag size (E-7 mm)	5.88	5.19	–0.69	6.13	6.06
10	Maximum bag pressure	6.63	5.50	–0.88	5.00	6.38

(b) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
2	Steering column rotation	(3)	(7.313)	Pooled		
3	Steering column crush stiffness	(3)	(8.438)	Pooled		
4	Knee bolster stiffness	(3)	(5.125)	Pooled		
5	Knee bolster location	3	9.063	3.021	1.631	4.65
6	Inflation rate	3	10.438	3.479	1.879	6.48
7	Development time	(3)	(7.938)	Pooled		
8	Vent size	3	9.563	3.188	1.721	5.32
9	Bag size (E-7 mm)	(3)	(4.438)	Pooled		
10	Maximum bag pressure	3	11.125	3.708	2.002	7.39
All other/error		19	35.19		1.85	76.17
Total:		31	75.38			100.00

Note: Insignificant factorial effects are pooled as shown ( ).

Table 9-20. Airbag design study (Optimum and confidence interval)—Example 9-7

(a) Estimate of Optimum Condition of Design/Process:
For Smaller is Better Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Knee bolster location	100	1	–0.8750
Inflation rate	Rate 4	4	–0.0625
Vent size	1300 mm	3	–0.7500
Maximum bag pressure	P3	3	–0.8125
Contribution from all factors (total)			–2.5000
Current grand average of performance			5.8125
Expected result at optimum condition			3.3125

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

(b) Confidence Interval

Computing F function for 1 and 19 at 90% confidence level.
Confidence Interval (C.I.) is expressed as:

$$\text{C.I.} = \sqrt{\frac{F(1, n_2) V_e}{N_e}}$$

where $F(n_1, n_2)$ = computed value of F with $n_1 = 1$, n_2 = error DOF at a desired confidence level

V_e = error variance
N_e = effective number of replications
Based on: $F = 2.600$, $n_1 = 1$, $n_2 = 19$, $V_e = 1.851974$,
$N_e = 2.461539$ [from 32/(1+12)]

The confidence interval C.I. = ±1.3

factors, each of which had two alternatives. The performance of the structure was measured in terms of the deformation under a drop silo test. For the test variability, three samples at each configuration were tested.

Solution—Example 9-9

The factor descriptions and the analyses are shown in Tables 9-22 and 9-23. The design and the analysis are straightforward. The analysis utilizes an S/N ratio with the “nominal is best” quality characteristic.

Table 9-21. Study of transmission control cable adjustment (Design and modified OA)—Example 9-8

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Adjuster type/source	Type 1	Type 2	Type 3	Type 4
2	(Used with column 1)				
3	(Used with column 1)				
4	Bracket deflection	Low	High		
5	Adjuster load	Low	High		
6	Cable elasticity	1 × 19	7 × 7		
7	Interaction 8 × 15	N/A			
8	Speed of adjuster	Low	High		
9	(Unused)				
10	(Unused)				
11	Interaction 4 × 15	N/A			
12	Interaction 4 × 15	N/A			
13	(Unused)				
14	(Unused)				
15	Adjusting torque	Low	High		

Note: A four-level factor and three interactions.
Objective: Determine best setting/adjustment.
Characteristic: Nominal is best (deviation from nominal measured).

(b) L_{16} Orthogonal Array Used for Experiment

COLUMN TRIAL	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Trial 1	1	0	0	1	1	1	1	1	0	0	1	1	0	0	1
Trial 2	1	0	0	1	1	1	1	2	0	0	2	2	0	0	2
Trial 3	1	0	0	2	2	2	2	1	0	0	1	2	0	0	2
Trial 4	1	0	0	2	2	2	2	2	0	0	2	1	0	0	1
Trial 5	2	0	0	1	1	2	2	1	0	0	2	2	0	0	2
Trial 6	2	0	0	1	1	2	2	2	0	0	1	2	0	0	1
Trial 7	2	0	0	2	2	1	1	1	0	0	2	1	0	0	1
Trial 8	2	0	0	2	2	1	1	2	0	0	1	2	0	0	2
Trial 9	3	0	0	1	2	1	2	1	0	0	2	2	0	0	2
Trial 10	3	0	0	1	2	1	2	2	0	0	1	1	0	0	1
Trial 11	3	0	0	2	1	2	1	1	0	0	2	2	0	0	1
Trial 12	3	0	0	2	1	2	1	2	0	0	1	2	0	0	2
Trial 13	4	0	0	1	2	2	1	1	0	0	1	1	0	0	1
Trial 14	4	0	0	1	2	2	1	2	0	0	2	2	0	0	2
Trial 15	4	0	0	2	1	1	2	1	0	0	2	2	0	0	2
Trial 16	4	0	0	2	1	1	2	2	0	0	1	1	0	0	1

Table 9-22. Front structure crush characteristics (Design and data)—Example 9-9

(a) Design Factors and Their Levels

COLUMN	FACTORS	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4
1	Lower rail section	Current	Proposed		
2	Upper rail geometry	Open section	Closed section		
3	Cross member	Present	Reinforced		

Note: Three two-level factors studied.
Objective: Determine best design for barrier crush.
Characteristic: Nominal is best (impact deformation).

(b) L_4 Orthogonal Array Used for Experiment

COLUMN / TRIAL	1	2	3
Trial 1	1	1	1
Trial 2	1	2	2
Trial 3	2	1	2
Trial 4	2	2	1

(c) Original Observations and Their S/N Ratios
Quality Characteristic: Nominal is Best

REPETITION / TRIAL	R_1	R_2	R_3	R_4	R_5	R_6	S/N
1	12.00	14.00	11.00				–6.37
2	18.00	16.00	15.00				–8.46
3	14.00	15.00	15.00				–1.76
4	19.00	18.00	15.00				–11.47

Example 9-10

Electronic Connector Spring Disengagement Force Study

A manufacturer of precision electronic switch assemblies was experiencing high rejects with one of its connectors. This connector consists of insertion of a solid, screw-machined pin into a flexible sleeve. The design created a compliant sleeve to generate sufficient spring force between the sleeve and the gold-plated, stamped metal pin similar to that shown in Figure 9-1. The plant has been producing the pin for several years. But recently, for some causes unknown, there has been higher than acceptable vari-

Table 9-23. Front structure crush characteristics (Main effects, ANOVA, and optima)—Example 9-9

(a) Main Effects

COLUMN	FACTOR	LEVEL 1	LEVEL 2	$L_2 - L_1$	LEVEL 3	LEVEL 4
1	Lower rail section	–7.42	–4.86	2.56	00.00	00.00
2	Upper rail section	–2.30	–9.97	–7.66	00.00	00.00
3	Cross member	–8.92	–3.35	5.57	00.00	00.00

(b) ANOVA Table

COLUMN	FACTOR	DOF	SUM OF SQUARES	VARIANCE	F	PERCENT
1	Lower rail section	1	6.554	6.554		6.81
2	Upper rail section	1	58.676	58.676		60.96
3	Cross member	1	31.025	31.025		32.23
All other/error		0				
Total:		3	96.250			100.00

(c) Estimate of Optimum Condition of Design/Process:
For Nominal is Best Characteristic

FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
Lower rail section	Proposed design	2	1.2800
Upper rail section	Open section	1	3.8300
Cross member	Reinforced design	2	2.7850
Contribution from all factors (total)			7.8950
Current grand average of performance			–6.1350
Expected result at optimum condition			1.7600

This estimate includes only those variables that have a significant contribution; that is, pooled variables are excluded from the estimate. Estimates may also be made with variables of choice.

ability. Precise automation and attention to detail in tool cutting fail to improve the situation. To seek a permanent solution, the production team launched a Taguchi experimental design study to optimize the process and reduce variability.

The project team dedicated an entire day to experiment planning to agree on the objective and identify factors. By consensus, the team selected seven factors from a "Paretoized" list of more than a dozen qualified factors. The production team also agreed that the spring disengagement force, which is a key characteristic for the part, will be used to measure the results.

Figure 9-1. Electronic connector switch component—Example 9-10

Solution—Example 9-10

The experiment was designed using a standard L-8 array by assigning seven two-level factors in the order shown (Table 9-24). Five sets of samples, with multiple fabricated parts in each set, were tested in each of the eight trial conditions. Description of an example trial condition (#3) is shown in Table 9-25. Upon completion of the tests, results (Table 9-26) were analyzed using S/N of results for the "bigger is better" quality characteristic. Computer software [7] was used to perform the analysis and draw conclusions.

Table 9-24. Study factors and their levels—Example 9-10

FACTOR DESCRIPTION	LEVEL 1	LEVEL 2
C: Machine setup	Nominal	With bushing
A: Spring gap	0.0185 in.	0.0215 in.
B: Crimp design	Current design	New design
D: Sleeve ID	0.050 in.	0.0507 in.
E: Spring contact radius	0.017 in.	0.022 in.
F: Metal hardness (spring, source)	Brush	NGK
G: Spring OD	0.0495 in.	0.0503 in.
Note: Interaction ignored and noise considered random.		

Table 9-25. Description of an example trial condition (3 of 8)—Example 9-10

Trial condition 3 (random order for running this trial is 5)		
FACTOR DESCRIPTION	LEVEL DESCRIPTION	LEVEL 1
C: Machine setup	Nominal	1
A: Spring gap	0.0215 in.	2
B: Crimp design	New design	2
D: Sleeve ID	0.050 in.	1
E: Spring contact radius	0.017 in.	1
F: Metal hardness (spring, source)	NGK	2
G: Spring OD	0.0503 in.	2

Table 9-26. Experimental results and S/N for trials (Bigger is better)—Example 9-10

CONDITIONS TRIAL	SAMPLE #1	SAMPLE #2	SAMPLE #3	SAMPLE #4	SAMPLE #5	SAMPLE #6	S/N RATIO
1	1.57	1.69	1.685	1.74	1.821		4.584
2	3.335	3.425	3.62	2.815	2.773		9.933
3	1.991	2.036	2.428	2.521	3.037		7.31
4	1.27	1.295	1.303	1.29	1.192		2.062
5	3.275	3.735	4.167	4.132	2.915		10.982
6	1.288	1.256	1.342	1.286	1.277		2.204
7	2.091	1.986	1.927	1.925	1.97		5.92
8	1.348	1.5	1.425	1.345	1.418		2.945
						Avg. =	5.742
All results: Avg. = 2.111, Std. dev. = .89							

Basic computations of factor average effects (main effects), potential two-factor interactions and ANOVA are shown in Tables 9-27 through 9-29 and Figure 9-2. Based on factor effects, the optimum condition and the expected performance in original units (force in oz.) are shown in Table 9-30 and Figure 9-3. Assuming the performance is on target, the variability reduction and expected cost savings can also be estimated (Figure 9-4). The following observations are derived from the analysis:

Table 9-27. Average effects of factors (S/N for bigger is better)—Example 9-10

COLUMN #/FACTORS	LEVEL 1	LEVEL 2	$L_2 - L_1$
1 C: Machine setup	5.972	5.513	–.46
2 A: Spring gap	6.926	4.559	–2.367
3 B: Crimp design	5.846	5.639	–.207
4 D: Sleeve ID	7.199	4.286	–2.914
5 E: Spring contact radius	4.261	7.224	2.963
6 F: Metal hardness (spring, source)	5.143	6.342	1.198
7 G: Spring OD	3.692	7.792	4.099

Table 9-28. Relative strength of presence of interaction and factor levels—Example 9-10

#	INTERACTING FACTOR PAIRS (ORDER BASED ON SI)	COLUMNS	SI (%)	COL.	OPT.
1	C: Machine setup × B: Crimp design	1 × 3	83.74	2	[1,1]
2	C: Machine setup × F: Metal hardness	1 × 6	77.38	7	[1,2]
3	B: Crimp design × F: Metal hardness	3 × 6	71.2	5	[1,2]
4	B: Crimp design × D: Sleeve ID	3 × 4	58.46	7	[2,1]
5	A: Spring gap × E: Spring c. rad.	2 × 5	58.04	7	[1,2]
6	A: Spring gap × F: Metal hardness	2 × 6	55.17	4	[1,1]
7	C: Machine setup × D: Sleeve ID	1 × 4	50.43	5	[2,1]
8	C: Machine setup × E: Spring c. rad.	1 × 5	49.56	4	[2,2]
9	D: Sleeve ID × F: Metal hardness	4 × 6	44.82	2	[1,1]
10	A: Spring gap × G: Spring OD	2 × 7	41.95	5	[1,2]
11	B: Crimp design × G: Spring OD	3 × 7	41.53	4	[2,2]

Table 9-29. Relative factor influences (from ANOVA)—Example 9-10

COLUMN #/ FACTORS	DOF (*f*)	SUM OF SQUARES (*S*)	VARIANCE (*V*)	F RATIO (*F*)	PURE SUM (*S′*)	PERCENT *P* (%)
1 C: Machine setup	(1)	(.421)		Pooled	(CL = 100%)	
2 A: Spring gap	1	11.2	11.2	44.193	10.947	13.231
3 B: Crimp design	(1)	(.085)		Pooled	(CL = *NC*)	
4 D: Sleeve ID	1	16.971	16.971	66.959	16.717	20.205
5 E: Spring c. rad.	1	17.565	17.565	69.304	17.311	20.923
6 F: Metal hardness	1	2.872	2.872	11.332	2.618	3.165
7 G: Spring OD	1	33.621	33.621	132.656	33.368	40.33
Other/Error	2	.506	.253			2.146
Total:	7	82.738				100.00%

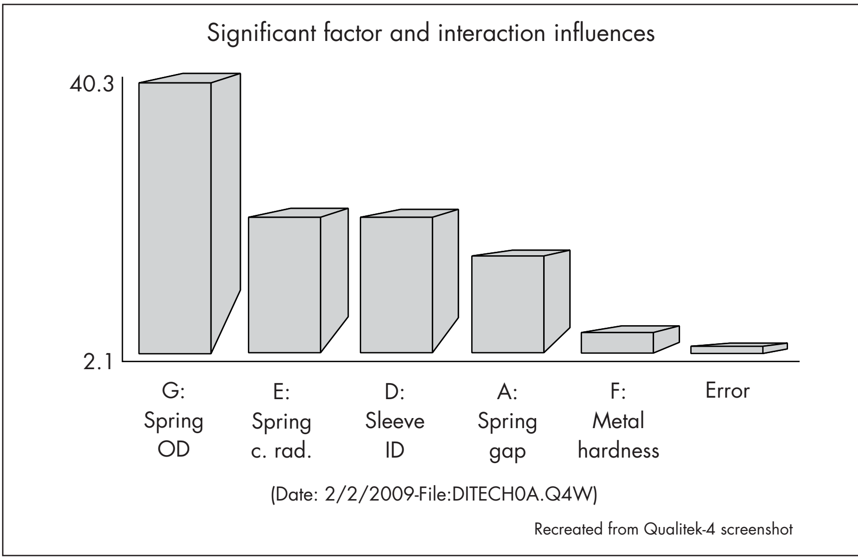


Figure 9-2. Bar graph of relative factor influences (from ANOVA)—Example 9-10

Table 9-30. Optimum condition and performance (S/N)—Example 9-10

COLUMN #/FACTORS	LEVEL DESCRIPTION	LEVEL	CONTRIBUTION
2 A: Spring gap	0.0185 in.	1	1.183
4 D: Sleeve ID	0.050 in.	1	1.456
5 E: Spring contact radius	0.022 in.	2	1.481
6 F: Metal hardness (spring, source)	NGK	2	.599
7 G: Spring OD	0.0503 in.	2	2.05
Total contribution from all factors			6.769
Current grand average of performance			5.742
Expected result at optimum condition			12.511

- Most influential factor is G: Spring OD (Larger diameter is better for spring force, Table 9-27). Factor *B:* Crimp Design has negligible effect on Spring Force.
- Factors with the most influence on *average* of Disengagement Force are: G: Spring OD, E: Spring Contact Radius,

Data Type: S/N Ratio QC Type: Bigger is Better

Estimate of expect results from S/N ratio

S/N = –10 Log (MSD) = 12.511

or MSD = 10^[–(S/N)/10] = 0.056092

where

MSD = [(1/y1)^2 + (1/y2)^2 + ... + (1/yn)^2]/n

= [Avg. (1/yi)^2] = 1/Yexp^2

or Yexp = SQR(1/MSD)

Expected performance in QC units

(or overall evaluation criteria) is:

Yexp = 4.222 QC units

(Based on S/N = 12.511 at optimum)

Recreated from Qualitek-4 screenshot

Figure 9-3. Expected performance at optimum in original units—Example 9-10

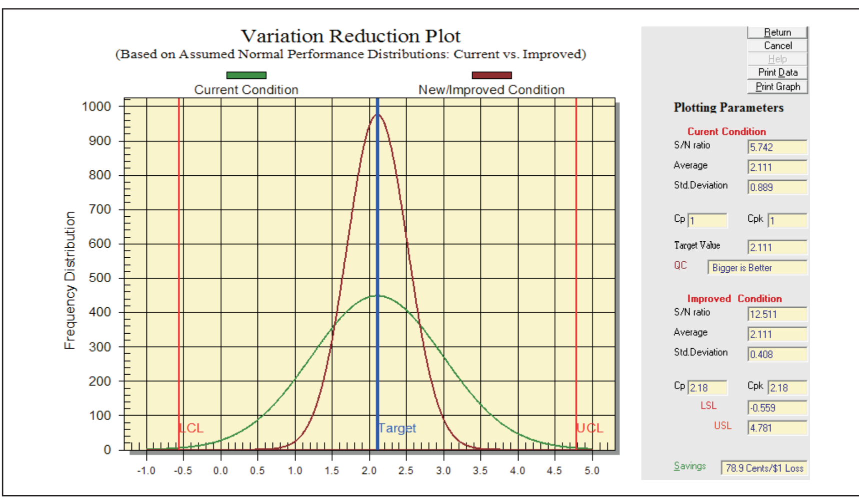


Figure 9-4. Plot showing variability and cost improvements—Example 9-10

Table 9-31. Solution design and expected performance—Example 9-10

OPTIMUM DESIGN	
C: Machine setup	With bushing
A: Spring gap	0.0185 in.
B: Crimp design	Either current or new design
D: Sleeve ID	0.050 in.
E: Spring contact radius	0.022 in.
F: Metal hardness (spring, source)	NGK
G: Spring OD	Lower end of 0.0499–0.0503 in.
Expected performance: Average of disengagement force in above design expected to be 4.22 oz. of spring force.	

D: Sleeve ID, and A: Spring Gap (in descending order of influence, Table 9-29).

- Factors that have the *least influence* are: B: Crimp Design, C: Machine Setup, and F: Metal Hardness. Levels for these factors are prescribed based on lower cost and ease of assembly.
- Design the most favorable DESIGN for higher average of Spring Force is as shown in Table 9-31. The recommended design was confirmed and was found to consistently deliver 70% higher average spring force than that before the investigation.

For more examples with S/N analysis, noise factors, and OEC, readers are referred to [2–5, 8].

ABBREVIATIONS AND SYMBOLS

A, B, ...	variables used in the design of an experiment
A_i	sum of observations under condition A_i (i = 1, 2, 3, ...)
$\bar{A}_i$	average of observations under condition A_i
C.I.	confidence interval
DOE	design of experiments
e	experimental error
f, n	degrees of freedom
F	variance ratio
k	a constant used in the expression for loss function
L	the Taguchi loss function
L_8	an orthogonal array that has eight experiments
LSL	lower specification limit
MSD	mean square deviation
N	number of experiments
OA	orthogonal array, L_4, L_8, L_{16}, and so on
P	percent contribution of a variable
S	sum of squares
S'	net/pure sum of squares
S/N	signal-to-noise ratio
T	sum of all observations
USL	upper specification limit
V	variance (mean square, S/f)
Y	results measured in terms of quality characteristics; for example, cost, weight, length, surface finish.
α	level of significance (among other usage)
μ	population mean
σ	population standard deviation
σ^2	population variance

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TO OUR GRANDSON, CIARAN, AND OUR GRANDDAUGHTER, KAMALA

GLOSSARY

ANOVA (Analysis of Variance)
An analysis of variance is a table of information that displays the contributions of each factor.

Controllable Factor
A design variable that is considered to influence the response and is included in the experiment. Its level can be controlled by the experimenter.

Design of Experiment
A systematic procedure to lay out the factors and conditions of an experiment. Taguchi employs specific partial factorial arrangements (orthogonal arrays) to determine the optimum experiment design.

Factorial Experiment
A systematic procedure in which all controllable factors except one are held constant as the variable factor is altered discretely or continuously.

Error
Amount of variation in the response caused by factors other than controllable factors included in the experiment.

Inner Array
Describes the combination of control factors and layout of the design of experiment.

Interaction
Two factors are said to have interaction with each other if the influence of one on the response function is dependent on the value of the other.

Linear Graph
A graphical representation of relative column locations of factors and their interactions. Linear graphs were developed by Dr. Taguchi to assist in assigning different factors to columns of the orthogonal array.

Loss Function
A mathematical expression proposed by Dr. Taguchi to quantitatively determine the additional cost to society caused by the lack of quality in a product. This additional cost is viewed

as a loss to society and is expressed as a direct function of the mean square deviation from the target value.

Noise Factors

Factors that have an influence over a response but cannot be controlled in actual applications. There are three types:

Outer noise: Consists of environmental conditions such as humidity temperature, operators, and so on.

Inner noise: Deterioration of machines, tools, and parts.

Between-product noise: Variation from piece to piece.

Off-Line Quality Control

The quality enhancement efforts in activities before production, such as, upstream planning, R&D, system design, parametric design, tolerance design, loss function, and so on.

Orthogonal Array (OA)

A set of tables used to determine the least number of experiments and their conditions. "Orthogonal" means balanced.

Outer Array

An orthogonal array that is used to define the conditions for the repetitions of the inner array to measure the effects of various noise factors. An experiment with outer arrays will reduce product variability and sensitivity to noise factors.

Parameter Design

Used to design a product by selecting the optimum condition of parameter levels so that the product is least sensitive to noise factors.

Quality Characteristic

Measures the performance of a product or a process under study. For example, for a plastic molding process, this could be the strength of the molded piece; for a cake, this could be a combination of taste, shape, and moistness.

Response

A quantitative value of the measured quality characteristic, for example, stiffness, weight, flatness.

Robustness

Describes a condition in which a product or process is least influenced by the variation of individual factors. To become robust is to become less sensitive to variations.

Signal Factor

A factor that influences the average value but not the variability in response.

S/N (Signal-to-Noise) Ratio

Ratio of the power of a signal to the power of the noise (error). A high S/N ratio means that there is high sensitivity with the least error of measurement. In Taguchi analysis using S/N ratios, a higher value is always desirable regardless of the quality characteristic.

System Design

The design of a product or process using special Taguchi techniques.

Taguchi Design

A methodology to increase quality by optimizing system design, parameter design, and tolerance design. This text deals with system design.

Target Value

A value that a product is expected to possess. Most often this value is different from what a single unit actually exhibits. For a 9-volt transistor battery, the target value is 9 volts.

Tolerance Design

A sophisticated version of parametric design that is used to optimize tolerance, reduce costs, and increase customer satisfaction.

Variables (or Factors or Parameters)

These words are used synonymously to indicate the controllable factors in an experiment. In the case of a plastic molding experiment, molding temperature, injection pressure, set time, and so on, are the factors.

Variation Reduction

Variation in the output of a process produces nonuniformity in the product and is perceived as an important criteria for quality. Reduced variation increases customer satisfaction and reduces warranty cost arising from variation. To achieve better quality, a product must perform optimally and should have less variation around the desired critical characteristic for quality.

WHAT READERS ARE SAYING...

"...a clear, step-by-step guide to the Taguchi design of experiments method. The careful descriptions, calculations and examples demonstrate the versatility of these practical and powerful tools."

— Fred Schenkelberg, consultant, FMS Reliability, Los Gatos, Calif.

"Dr. Roy presents the theory and relates it to practical examples, explaining difficult concepts in an understandable manner. This is an easy-to-read, right-on-the-mark guide to understanding and applying Taguchi robust design and DOE. Readers will find these techniques extremely useful, practical and easily applied to the daily job."

— George Li, process improvement manager, Research In Motion, Waterloo, Ont.

"The book has a detailed discussion of Taguchi methods that are not covered in great detail in many books on DOE."

— Frederick H. Long, president, Spectroscopic Solutions, LLC, Randolph, N.J.

"Dr. Roy's name is instantly associated with Taguchi methodologies in the manufacturing industries. His skill set is also being recognized for project management instruction. The new edition includes more easy-to-follow descriptions and examples."

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"The author is able to explain concepts in a very knowledgeable yet down-to-earth and systematic manner. The material is very well organized."

— Kush Shah, manager, alternative propulsion technology quality, General Motors, LLC, Pontiac, Mich.

"This book is a valuable introductory text in Taguchi methods with a number of illustrative examples and case studies that make the concepts clearer than books with theory only."

— Dr. R. Mahalinga Iyer, senior lecturer, Queensland University of Technology, Brisbane, Queensland, Australia

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Appendix A

ORTHOGONAL ARRAYS, TRIANGULAR TABLES, AND LINEAR GRAPHS

*Table A-1. Common orthogonal arrays***

ARRAY	NUMBER OF FACTORS	NUMBER OF LEVELS
L_4 (2^3)	3	2
L_8 (2^7)	7	2
L_{12} (2^{11})	11	2
L_{16} (2^{15})	15	2
L_{32} (2^{31})	31	2
L_9 (3^4)	4	3
*L_{18} (2^1, 3^7)	1 and 7	2 3
L_{27} (3^{13})	13	3
L_{16} (4^5)	5	4
*L_{32} (2^1, 4^9)	1 and 9	2 4
L_{64} (4^{21})	21	4

* Mixed-level arrays
** Orthogonal arrays from G. Taguchi and S. Konishi, *Orthogonal Arrays and Linear Graphs—Tools for Quality Engineering*, Dearborn, MI: American Supplier Institute, Inc., 1987.

*Table A-2. Orthogonal arrays L_4 and L_8 (two-level)**

(a)

COLUMN / CONDITION	L_4 (2^3) 1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

(b)

	L_8 (2^7) 1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

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*Table A-3. Orthogonal arrays L_{12} and L_{16} (two-level)**

$L_{12}(2^{11})$

NO.	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	1	2	2	2	1

The $L_{12}(2^{11})$ is a specially designed array in that interactions are distributed more or less uniformly to all columns. There is no linear graph for this array. It should not be used to analyze interactions. The advantage of this design is its capability to investigate 11 main effects, making it a highly recommended array.

$L_{16}(2^{15})$

NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

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Figure A-1. Linear graph for L_4

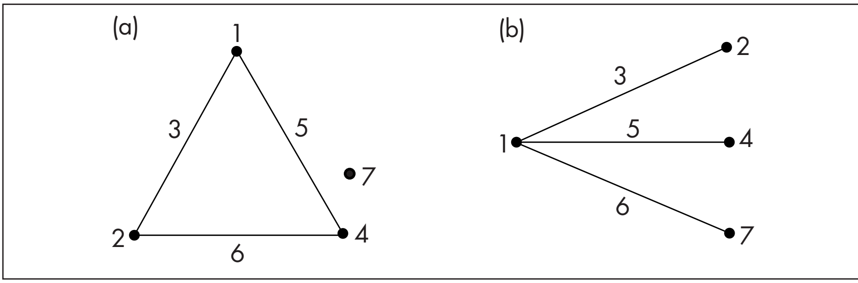

Figure A-2. Linear graphs for L_8

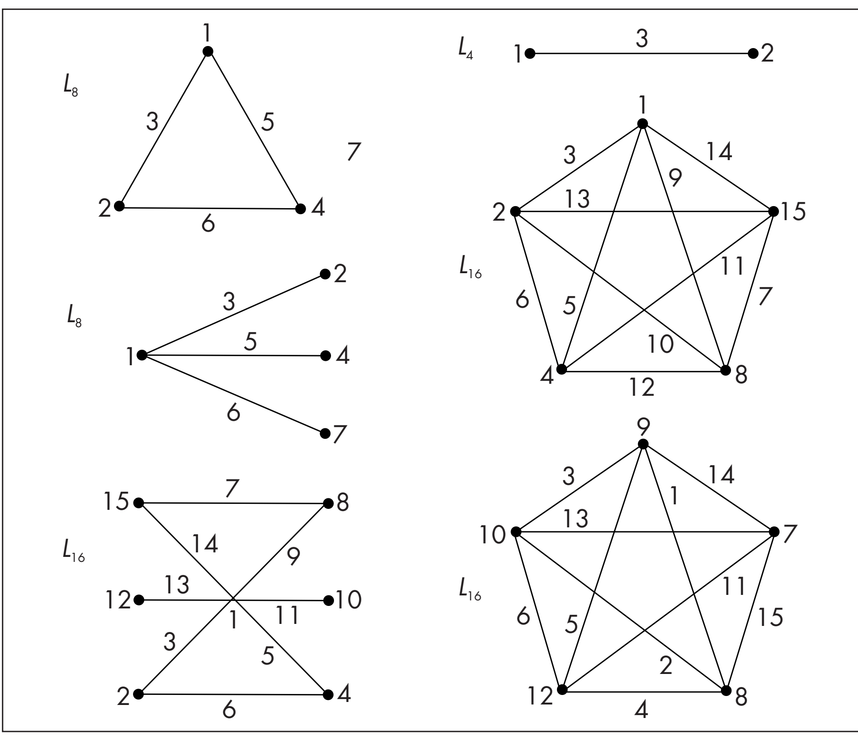

Figure A-3. Linear graphs for two-level orthogonal arrays

*Table A-4. Orthogonal arrays L_{32} (two-level, 31 factors)**

COLUMN	L_{32} (2^{31})																													
CONDITION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
5	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
6	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
7	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
8	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
9	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
10	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
11	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
12	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
13	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
14	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
15	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
16	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
17	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
18	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
19	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1
20	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
21	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1

22	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
23	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
24	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
25	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
26	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
27	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	1
28	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
29	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
30	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
31	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
32	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2

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*Table A-5(a). Orthogonal arrays L_{64} (two-level)**

COLUMN / CONDITION	L_{64} (2^{63}) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
5	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
6	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
7	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
9	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
10	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
11	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
12	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
13	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
14	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
15	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
16	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
17	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
18	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
19	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
20	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
21	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1

22	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
23	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
24	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
25	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
26	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
27	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
28	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
29	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
30	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
31	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
32	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
33	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
34	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2
35	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
36	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
37	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
38	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
39	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
40	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
41	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
42	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
43	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
44	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
45	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
46	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2

(continued)

Table A-5(a). Orthogonal arrays L_{64} (two-level) (continued)*

COLUMN / CONDITION	L_{64} (2^{63}) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
47	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
48	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
49	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
50	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
51	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
52	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
53	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
54	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
55	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
56	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
57	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
58	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
59	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
60	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
61	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
62	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
63	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2
64	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2

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Table A-5(b). Orthogonal array L_{64} [continues Table A-5(a)]*

L_{64} (2^{63})																															
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1
2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2
2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1
1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	2	2	2	1	1
2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2

(continued)

Table A-5(b). Orthogonal array L_{64} (continued)*

L_{64} (2^{63})																															
32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2
2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2
2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1
2	2	1	1	1	1	2	2	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1
2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	1	2	1	2	1	2
1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	1
1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2
2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1

1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
2	1	2	1	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1	1	1	2	1	2	1	2	1	2	2	1	2	1
1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1
2	1	2	1	1	2	1	2	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1
2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
1	2	2	1	2	1	1	2	1	2	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1
2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1
2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2
1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1	2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2
2	1	1	2	1	2	2	1	1	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

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*Table A-6. Triangular table for two-level orthogonal arrays**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
(1)	3	2	5	4	7	6	9	8	11	10	13	12	15	14	17	16	19	18	21	20	23	22	25	24	27	26	29	28	31	30
	(2)	1	6	7	4	5	10	11	8	9	14	15	12	13	18	19	16	17	22	23	20	21	26	27	24	25	30	31	28	29
		(3)	7	6	5	4	11	10	9	8	15	14	13	12	19	18	17	16	23	22	21	20	27	26	25	24	31	30	29	28
			(4)	1	2	3	12	13	14	15	8	9	10	11	20	21	22	23	16	17	18	19	28	29	30	31	24	25	26	27
				(5)	3	2	13	12	15	14	9	8	11	10	21	20	23	22	17	16	19	18	29	28	31	30	25	24	27	26
					(6)	1	14	15	12	13	10	11	8	9	22	23	20	21	18	19	16	17	30	31	28	29	26	27	24	25
						(7)	15	14	13	12	11	10	9	8	23	22	21	20	19	18	17	16	31	30	29	28	27	26	25	24
							(8)	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	16	17	18	19	20	21	22	23
								(9)	3	2	5	4	7	6	25	24	27	26	29	28	31	30	17	16	19	18	21	20	23	22
									(10)	1	6	7	4	5	26	27	24	25	30	31	28	29	18	19	16	17	22	23	20	21
										(11)	7	6	5	4	27	26	25	24	31	30	29	28	19	18	17	16	23	22	21	20
											(12)	1	2	3	28	29	30	31	24	25	26	27	20	21	22	23	16	17	18	19
												(13)	3	2	29	28	31	30	25	24	27	26	21	20	23	22	17	16	19	18
													(14)	1	30	31	28	29	26	27	24	25	22	23	20	21	18	19	16	17
														(15)	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16
															(16)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
																(17)	3	2	5	4	7	6	9	8	11	10	13	12	15	14
																	(18)	1	6	7	4	5	10	11	8	9	14	15	12	13
																		(19)	7	6	5	4	11	10	9	8	15	14	13	12
																			(20)	1	2	3	12	13	14	15	8	9	10	11
																				(21)	3	2	13	12	15	14	9	8	11	10
																					(22)	1	14	15	12	13	10	11	8	9
																						(23)	15	14	13	12	11	10	9	8
																							(24)	1	2	3	4	5	6	7
																								(25)	3	2	5	4	7	6
																									(26)	1	6	7	4	5
																										(27)	7	6	5	4
																											(28)	1	2	3
																												(29)	3	2
																													(30)	1

*Table A-7. Orthogonal arrays (three-level, L_9 and L_{18})**

(a)

COLUMN / CONDITION	$L_9(3^4)$ 1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

(b)

COLUMN / CONDITION	$L_{18}(2^1 \times 3^7)$ 1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

Note: Like the $L_{12}(2^{11})$, this is a specially designed array. An interaction is built in between the first two columns. This interaction information can be obtained without sacrificing any other column. Interactions between three-level columns are distributed more or less uniformly to all the other three-level columns, which permits investigation of main effects. Thus, it is a highly recommended array for experiments.

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Table A-8. Orthogonal arrays (three-level, L_{27})*

COLUMN / CONDITION	$L_{27}(3^{13})$												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

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Table A-9. Triangular table for three-level orthogonal arrays*

1	2	3	4	5	6	7	8	9	10	11	12	13
(1)	3	2	2	6	5	5	9	8	8	12	11	11
	4	4	3	7	7	6	10	10	9	13	13	12
	(2)	1	1	8	9	10	5	6	7	5	6	7
		4	3	11	12	13	11	12	13	8	9	10
		(3)	1	9	10	8	7	5	6	6	7	5
			2	13	11	11	12	13	11	10	8	9
			(4)	10	8	9	6	7	5	7	5	6
				12	13	11	13	11	12	9	10	8
				(5)	1	1	2	3	4	2	4	3
					7	6	11	13	12	8	10	9
					(6)	1	4	2	3	3	2	4
						5	13	12	11	10	9	8
						(7)	3	4	2	4	3	2
							12	11	13	9	8	10
							(8)	1	1	2	3	4
								10	9	5	7	6
								(9)	1	4	2	3
									8	7	6	5
									(10)	3	4	2
										6	5	7
										(11)	1	1
											13	12
											(12)	1
												11

*Table A-10. Orthogonal arrays (four-level)**

	$L_{16}(4^5)$				
NO.	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	2	1	2	3	4
6	2	2	1	4	3
7	2	3	4	1	2
8	2	4	3	2	1
9	3	1	3	4	2
10	3	2	4	3	1
11	3	3	1	2	4
12	3	4	2	1	3
13	4	1	4	2	3
14	4	2	3	1	4
15	4	3	2	4	1
16	4	4	1	3	2

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Table A-11. Orthogonal arrays (two-level and four-level)*

	$L_{32}(2^1 \times 4^9)$									
NO.	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3	3	3
4	1	1	4	4	4	4	4	4	4	4
5	1	2	1	1	2	2	3	3	4	4
6	1	2	2	2	1	1	4	4	3	3
7	1	2	3	3	4	4	1	1	2	2
8	1	2	4	4	3	3	2	2	1	1
9	1	3	1	2	3	4	1	2	3	4
10	1	3	2	1	4	3	2	1	4	3
11	1	3	3	4	1	2	3	4	1	2
12	1	3	4	3	2	1	4	3	2	1
13	1	4	1	2	4	3	3	4	2	1
14	1	4	2	1	3	4	4	3	1	2
15	1	4	3	4	2	1	1	2	4	3
16	1	4	4	3	1	2	2	1	3	4
17	2	1	1	4	1	4	2	3	2	3
18	2	1	2	3	2	3	1	4	1	4
19	2	1	3	2	3	2	4	1	4	1
20	2	1	4	1	4	1	3	2	3	2
21	2	2	1	4	2	3	4	1	3	2
22	2	2	2	3	1	4	3	2	4	1
23	2	2	3	2	4	1	2	3	1	4
24	2	2	4	1	3	2	1	4	2	3
25	2	3	1	3	3	1	2	4	4	2
26	2	3	2	4	4	2	1	3	3	1
27	2	3	3	1	1	3	4	2	2	4
28	2	3	4	2	2	4	3	1	1	3
29	2	4	1	3	4	2	4	2	1	3
30	2	4	2	4	3	1	3	1	2	4
31	2	4	3	1	2	4	2	4	3	1
32	2	4	4	2	1	3	1	3	4	2

*Table A-12. Triangular table for four-level orthogonal arrays**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	3	2	2	2	7	6	6	6	11	10	10	10	15	14	14	14	19	18	18	18
(1)	4	4	3	3	8	8	7	7	12	12	11	11	16	16	15	15	20	20	19	19
	5	5	5	4	9	9	9	8	13	13	13	12	17	17	17	16	21	21	21	20
		1	1	1	10	11	12	13	6	7	8	9	6	7	8	9	6	7	8	9
	(2)	4	3	3	14	15	16	17	14	15	16	17	10	11	12	13	10	11	12	13
		5	5	4	18	19	20	21	18	19	20	21	18	19	20	21	14	15	16	17
			1	1	11	10	13	12	7	6	9	8	8	9	6	7	9	8	7	6
		(3)	2	2	16	17	14	15	17	16	15	14	13	12	11	10	12	13	10	11
			5	4	21	20	19	18	20	21	18	19	19	18	21	20	15	14	17	16
				1	12	13	10	11	8	9	6	7	9	8	7	6	7	6	9	8
			(4)	2	17	16	15	14	15	14	17	16	11	10	13	12	13	12	11	10
				3	19	18	21	20	21	20	19	18	20	21	18	19	16	17	14	15
					13	12	11	10	9	8	7	6	7	6	9	8	8	9	6	7
				(5)	15	14	17	16	16	17	14	15	12	13	10	11	11	10	13	12
					20	21	18	19	19	18	21	20	21	20	19	18	17	16	15	14
						1	1	1	2	3	4	5	2	5	3	4	2	4	5	3
					(6)	8	7	7	14	16	17	15	10	13	11	12	10	12	13	11
						9	9	8	18	21	19	20	18	20	21	19	14	17	15	16
							1	1	3	2	5	4	5	2	4	3	4	2	3	5
						(7)	6	6	17	15	14	16	12	11	13	10	13	11	10	12
							9	8	20	19	21	18	21	19	18	20	16	15	17	14
								1	4	5	2	3	3	4	2	5	5	3	2	4
							(8)	6	15	17	15	14	13	10	12	11	11	13	12	10
								7	11	18	20	19	19	21	20	18	17	14	16	15
									5	4	3	2	4	3	5	3	3	5	4	2
								(9)	16	14	15	17	11	12	10	13	12	10	11	13
									19	20	18	21	20	18	19	21	15	16	14	17
										1	1	1	2	4	5	3	2	5	3	4
									(10)	12	11	11	6	8	9	7	6	9	7	8
										13	13	12	18	21	19	20	14	16	17	15

(11)	1 10 13	1 10 12	4 9 20	2 7 19	3 6 21	5 8 18	5 8 17	2 7 15	4 9 14	3 6 16
	(12)	1 10 11	5 7 21	3 9 18	2 8 20	4 6 19	3 9 15	4 6 17	2 8 16	5 7 14
		(13)	3 8 19	5 6 20	4 7 18	2 9 21	4 7 16	3 8 14	5 6 15	2 9 17
			(14)	1 16 17	1 15 17	1 15 16	2 6 10	3 8 13	4 9 11	5 7 12
				(15)	1 14 17	1 14 16	3 9 12	2 7 11	5 6 13	4 8 10
					(16)	1 14 15	4 7 13	5 9 10	2 8 12	3 6 11
						(17)	5 8 11	4 6 12	3 7 10	2 9 13
							(18)	1 20 21	1 19 21	1 19 20
								(19)	1 18 21	1 18 20
									(20)	1 18 19

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Appendix B

TAGUCHI EXPERIMENT FLOW DIAGRAM AND *F*-TABLES

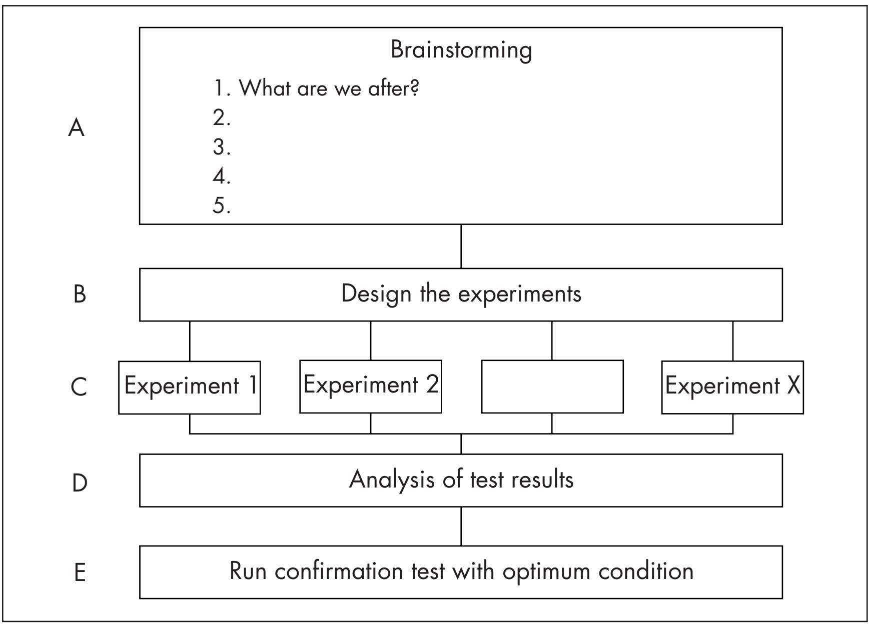

Figure B-1. A Taguchi experiment flow diagram

Table B-1. F-table $F_{.10}(f_1,f_2)$, 90% confidence

f_1 = number of degrees of freedom of numerator
f_2 = number of degrees of freedom of denominator

f_2 \ f_1	1	2	3	4	5	6	7	8	9
1	39.864	49.500	53.593	55.833	57.241	58.204	58.906	59.439	59.858
2	8.5263	9.0000	9.1618	9.2434	9.2926	9.3255	9.3491	9.3668	9.3805
3	5.5383	5.4624	5.3908	5.3427	5.3092	5.2847	5.2662	5.2517	5.2400
4	4.5448	4.3246	4.1908	4.1073	4.0506	4.0098	3.9790	3.9549	3.9357
5	4.0604	3.7797	3.6195	3.5202	3.4530	3.4045	3.3679	3.3393	3.3163
6	3.7760	3.4633	3.2888	3.1808	3.1075	3.0546	3.0145	2.9830	2.9577
7	3.5894	3.2574	3.0741	2.9605	2.8833	2.8274	2.7849	2.7516	2.7247
8	3.4579	3.1131	2.9238	2.8064	2.7265	2.6683	2.6241	2.5893	2.5612
9	3.3603	3.0065	2.8129	2.6927	2.6106	2.5509	2.5053	2.4594	2.4403
10	3.2850	2.9245	2.7277	2.6053	2.5216	2.4606	2.4140	2.3772	2.3473
11	3.2252	2.8595	2.6602	2.5362	2.4512	2.3981	2.3416	2.3040	2.2735
12	3.1765	2.8068	2.6055	2.4801	2.3940	2.3310	2.2828	2.2446	2.2135
13	3.1362	2.7632	2.5603	2.4337	2.3467	2.2830	2.2341	2.1953	2.1638
14	3.1022	2.7265	2.5222	2.3947	2.3059	2.2426	2.1931	2.1539	2.1220
15	3.0732	2.6952	2.4898	2.3614	2.2730	2.2081	2.1582	2.1185	2.0862
16	3.0481	2.6682	2.4618	2.3327	2.2438	2.1783	2.1280	2.0880	2.0553
17	3.0262	2.6446	2.4374	2.3077	2.2183	2.1524	2.1017	2.0613	2.0284
18	3.0070	2.6239	2.4160	2.2858	2.1958	2.1296	2.0785	2.0379	2.0047
19	2.9899	2.6056	2.3970	2.2663	2.1760	2.1094	2.0580	2.0171	1.9836
20	2.9747	2.5893	2.3801	2.2489	2.1582	2.0913	2.0397	1.9985	1.9649
21	2.9609	2.5746	2.3549	2.2333	2.1423	2.0751	2.0232	1.9819	1.9480
22	2.9486	2.5613	2.3512	2.2193	2.1279	2.0605	2.0084	1.9668	1.9327
23	2.9374	2.5493	2.3387	2.2065	2.1149	2.0472	1.9949	1.9531	1.9189
24	2.9271	2.5383	2.3274	2.1949	2.1030	2.0351	1.9826	1.9407	1.9063
25	2.9177	2.5283	2.3170	2.1843	2.0922	2.0241	1.9714	1.9292	1.8947
26	2.9091	2.5191	2.3075	2.1745	2.0822	2.0139	1.9610	1.9188	1.8841
27	2.9012	2.5106	2.2987	2.1655	2.0730	2.0045	1.9515	1.9091	1.8743
28	2.8939	2.5028	2.2906	2.1571	2.0645	1.9959	1.9427	1.9001	1.8652
29	2.8871	2.4955	2.2831	2.1494	2.0566	1.9678	1.9345	1.8918	1.8560
30	2.8807	2.4887	2.2761	2.1422	2.0492	1.9803	1.9269	1.8841	1.8498
40	2.8354	2.4404	2.2261	2.0909	1.9968	1.9269	1.8725	1.8289	1.7929
60	2.7914	2.3932	2.1774	2.0410	1.9457	1.8747	1.8194	1.7748	1.7380
120	2.7478	2.3473	2.1300	1.9923	1.8959	1.8238	1.7675	1.7220	1.6843
∞	2.7055	2.3026	2.0638	1.9449	1.8473	1.7741	1.7167	1.6702	1.6315

(continued)

10	12	15	20	24	30	40	60	120	∞
60.195	60.705	61.220	61.740	62.002	62.265	62.529	62.794	63.061	63.328
9.3916	9.4081	9.4247	9.4413	9.4496	9.4579	9.4663	9.4746	9.4829	9.4913
5.2304	5.2156	5.2003	5.1845	5.1764	5.1681	5.1597	5.1512	5.1425	5.1337
3.9199	3.8955	3.8689	3.8443	3.8310	3.8174	3.8036	3.7986	3.7753	3.7607
3.2974	3.2682	3.2380	3.2067	3.1905	3.1741	3.1573	3.1402	3.1228	3.1050
2.9369	2.9047	2.8712	2.8363	2.8183	2.8000	2.7812	2.7620	2.7423	2.7222
2.7025	2.6681	2.6322	2.5947	2.5723	2.5555	2.5351	2.5142	2.4928	2.4708
2.5380	2.5020	2.4642	2.4246	2.4041	2.3830	2.3614	2.3391	2.3162	2.2926
2.4163	2.3789	2.3396	2.2983	2.2768	2.2547	2.2320	2.2085	2.1843	2.1592
2.3226	2.2841	2.2435	2.2007	2.1784	2.1554	2.1317	2.1072	2.0818	2.0554
2.2482	2.2087	2.1671	2.1230	2.1000	2.0762	2.0516	2.0261	1.9997	1.9721
2.1878	2.1474	2.1049	2.0597	2.0360	2.0115	1.9861	1.9597	1.9323	1.9036
2.1376	2.0966	2.0532	2.0070	1.9827	1.9576	1.9315	1.9043	1.8759	1.8462
2.0954	2.0537	2.0095	1.9625	1.9377	1.9119	1.8852	1.8572	1.8280	1.7973
2.0593	2.0171	1.9722	1.9243	1.8890	1.8728	1.8454	1.8168	1.7867	1.7551
2.0281	1.9854	1.9399	1.8913	1.8656	1.8388	1.8108	1.7816	1.7507	1.7182
2.0009	1.9577	1.9117	1.8624	1.8362	1.8090	1.7805	1.7506	1.7191	1.6856
1.9770	1.9333	1.8868	1.8368	1.8103	1.7827	1.7537	1.7232	1.6910	1.6567
1.9557	1.9117	1.8647	1.8142	1.7873	1.7592	1.7298	1.6988	1.6659	1.6308
1.9367	1.8924	1.8449	1.7938	1.7667	1.7382	1.7083	1.6769	1.6433	1.6074
1.9197	1.8750	1.8272	1.7756	1.7481	1.7193	1.6890	1.6569	1.6228	1.5862
1.9043	1.8503	1.8111	1.7590	1.7312	1.7021	1.6714	1.6389	1.6042	1.5668
1.8903	1.8450	1.7964	1.7439	1.7159	1.6864	1.6554	1.6224	1.5871	1.5490
1.8775	1.8319	1.7831	1.7302	1.7019	1.6721	1.6407	1.6073	1.5715	1.5327
1.8548	1.8200	1.7708	1.7175	1.6890	1.6589	1.6272	1.5934	1.5570	1.5176
1.8550	1.8090	1.7596	1.7059	1.6771	1.6468	1.6147	1.5805	1.5437	1.5036
1.8451	1.7989	1.7492	1.6951	1.6662	1.6356	1.6032	1.5687	1.5313	1.4906
1.8359	1.7895	1.7395	1.6852	1.6560	1.6252	1.5925	1.5575	1.5198	1.4784
1.8274	1.7808	1.7306	1.6759	1.6465	1.6155	1.5825	1.5472	1.5090	1.4670
1.8195	1.7727	1.7223	1.6673	1.6377	1.6065	1.5732	1.5376	1.4989	1.4564
1.7627	1.7146	1.6624	1.6052	1.5741	1.5411	1.5056	1.4572	1.4248	1.3769
1.7070	1.6574	1.6034	1.5435	1.5107	1.4755	1.4373	1.3952	1.3476	1.2915
1.6524	1.6012	1.5450	1.4821	1.4472	1.4094	1.3676	1.3203	1.2646	1.1926
1.5987	1.5458	1.4871	1.4206	1.3832	1.3410	1.2951	1.2400	1.1686	1.0000

Table B-2. F-table $F_{.05}$ (f_1,f_2), 95% confidence

f_1 = number of degrees of freedom of numerator
f_2 = number of degrees of freedom of denominator

f_2 \ f_1	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.614	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8868	8.8452	8.8123
4	7.7086	6.9443	6.5914	6.3883	6.2560	6.1631	6.0942	6.0410	5.9988
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2830	4.2066	4.1468	4.0990
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8	5.3177	4.4590	4.0662	3.8378	3.6875	3.5806	3.5005	3.4381	3.3881
9	5.1174	4.2565	3.8626	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19	4.3808	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20	4.3513	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3661
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24	4.2597	3.4026	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26	4.2252	3.3690	2.9751	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2782	2.2229
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40	4.0848	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	1.1240
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2540	2.1665	1.0970	2.0401
120	3.9201	3.0718	2.6802	2.4472	2.2900	2.1750	2.0867	2.0164	1.9588
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

(continued)

10	12	15	20	24	30	40	60	120	∞
241.88	243.91	245.95	248.01	249.05	250.09	251.14	252.20	253.25	254.32
19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5265
5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6878	5.6581	5.6281
4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3984	4.3650
4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6688
3.6365	3.5747	3.5108	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
3.3472	3.2840	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
2.6021	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2230	2.1778	2.1307
2.5437	2.4753	2.4035	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9796	1.9302	1.8780
2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8895	1.8380	1.7831
2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8649	1.8128	1.7570
2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7897	1.7331
2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7307	1.6717
2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6377
2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000

Table B-3. F-table $F_{.025}(f_1,f_2)$, 97.5% confidence

f_1 = number of degrees of freedom of numerator
f_2 = number of degrees of freedom of denominator

f_2 \ f_1	1	2	3	4	5	6	7	8	9
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
2	38.506	39.000	39.165	39.248	39.298	39.331	39.355	39.373	39.387
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.540	14.473
4	12.218	10.649	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6810
6	8.8131	7.2598	6.5988	6.2272	5.9876	5.8197	5.6955	5.5996	5.5234
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8994	4.8232
8	7.5709	6.0595	5.4160	5.0526	4.8173	4.6517	4.5286	4.4332	4.3572
9	7.2093	5.7147	4.0781	4.7181	4.4844	4.3107	4.1971	4.1020	4.0260
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.7790
11	6.7241	5.2559	4.6300	4.2751	4.0440	3.8807	3.7586	3.6638	3.5879
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7293	3.6065	3.5118	3.4358
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.3880	3.3120
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093
15	6.1995	4.7650	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488
17	6.0420	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.0610	2.9849
18	5.9781	4.5597	3.9539	3.6083	3.3820	3.2209	3.0999	3.0053	2.9291
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8800
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.9740	2.7977
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9024	2.8077	2.7313
24	5.7167	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027
25	5.6864	4.2909	3.6843	3.3530	3.1287	2.9685	2.8478	2.7531	2.6766
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.8240	2.7293	2.6528
27	5.6331	4.2421	3.6472	3.3067	3.0628	2.9228	2.8021	2.7074	2.6309
28	5.6096	4.2205	3.6264	3.2863	3.0625	2.9027	2.7820	2.6872	2.6106
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.8840	2.7633	2.6686	2.5919
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.7460	2.6513	2.5746
40	5.4239	4.0510	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519
60	5.2857	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344
120	5.1524	3.8046	3.2270	2.8943	2.6740	2.5154	2.3948	2.2994	2.2217
∞	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136

(continued)

10	12	15	20	24	30	40	60	120	∞
968.63	976.71	984.87	993.10	997.25	1001.4	1005.6	1009.8	1014.0	1018.3
39.398	39.415	39.431	39.448	39.456	39.465	39.473	39.481	39.490	39.498
14.419	14.337	14.253	14.167	14.124	14.081	14.037	13.992	13.947	13.902
8.8439	8.7512	8.6565	8.5599	8.5109	8.4613	8.4111	8.3604	8.3092	8.2573
6.6192	6.5246	6.4277	6.3285	6.2780	6.2269	6.1751	6.1225	6.0693	6.0153
5.4613	5.3662	5.2687	5.1684	5.1172	5.0652	5.0125	5.9589	4.9045	4.8491
4.7611	4.6658	4.5678	4.4667	4.4150	4.3624	4.3089	4.2544	4.1989	4.1423
4.2951	4.1997	4.1012	3.9995	3.9472	3.8940	3.8398	3.7844	3.7279	3.6702
3.9639	3.8682	3.7694	3.6669	3.6142	3.5604	3.5055	3.4493	3.3918	3.3329
3.7168	3.6209	3.5217	3.4186	3.3654	3.3110	3.2554	3.1984	3.1399	3.0798
3.5257	3.4296	3.3299	3.2261	3.1725	3.1176	3.0613	3.0035	2.9441	2.8828
3.3736	3.2773	3.1772	3.0728	3.0187	2.9633	2.9063	2.8478	2.7874	2.7249
3.2497	3.1532	3.0527	2.9477	2.8932	2.8373	2.7797	2.7204	2.6590	2.5955
3.1469	3.0501	2.9493	2.8437	2.7888	2.7324	2.6742	2.6142	2.5519	2.4872
3.0602	2.9633	2.8621	2.7559	2.7006	2.6437	2.5850	2.5242	2.4611	2.3953
2.9862	2.8890	2.7875	2.6808	2.6252	2.5678	2.5085	2.4471	2.3831	2.3163
2.9222	2.8249	2.7230	2.6158	2.5598	2.5021	2.4422	2.3801	2.3153	2.2474
2.8664	2.7689	2.6667	2.5590	2.5027	2.4445	2.3842	2.3214	2.2558	2.1869
2.8173	2.7196	2.6171	2.5089	2.4523	2.3937	2.3329	2.2695	2.2032	2.1333
2.7737	2.6758	2.5731	2.4645	2.4076	2.3486	2.2873	2.2234	2.1562	2.0853
2.7348	2.6368	2.5338	2.4247	2.3675	2.3082	2.2465	2.1819	2.1141	2.0422
2.6998	2.6017	2.4984	2.3890	2.3315	2.2718	2.2097	2.1446	2.0760	2.0032
2.6682	2.5699	2.4665	2.3567	2.2989	2.2389	2.1763	2.1107	2.0415	1.9677
2.6396	2.5412	2.4374	2.3273	2.2693	2.2090	2.1460	2.0799	2.0099	1.9353
2.6135	2.5149	2.4110	2.3005	2.2422	2.1816	2.1183	2.0517	1.9811	1.9055
2.5895	2.4909	2.3867	2.2759	2.2174	2.1565	2.0928	2.0257	1.9545	1.8781
2.5675	2.4688	2.3644	2.2533	2.1946	2.1334	2.0693	2.0018	1.9299	1.8527
2.5473	2.4484	2.3438	2.2324	2.1735	2.1121	2.0477	1.9796	1.9072	1.8291
2.5286	2.4295	2.3248	2.2131	2.1540	2.0923	2.0276	1.9591	1.8861	1.8072
2.5112	2.4120	2.3072	2.1952	2.1359	2.0739	2.0089	1.9400	1.8664	1.7867
2.3882	2.2882	2.1819	2.0677	2.0069	1.9429	1.8752	1.8028	1.7242	1.6371
2.2702	2.1692	2.0613	1.9445	1.8817	1.8152	1.7440	1.6668	1.5810	1.4822
2.1570	2.0548	1.9450	1.8249	1.7597	1.6899	1.6141	1.5299	1.4327	1.3104
2.0493	1.9447	1.8326	1.7085	1.6402	1.5660	1.4835	1.3883	1.2684	1.0000

Table B-4. F-table $F_{.01}$ (f_1,f_2), 99% confidence

f_1 = Number of degrees of freedom of numerator
f_2 = Number of degrees of freedom of denominator

f_2 \ f_1	1	2	3	4	5	6	7	8	9
1	4052.2	4999.5	5403.3	5624.6	5763.7	5859.0	5928.3	5981.6	6022.5
2	98.503	99.000	99.166	99.249	99.299	99.332	99.356	99.374	99.388
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4	21.198	18.000	16.694	15.977	15.522	15.207	14.986	14.799	14.659
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6	13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1016	7.9761
7	12.246	9.5466	8.4513	7.8467	7.4604	7.1914	6.9928	6.8401	6.7188
8	11.259	8.6491	7.5910	7.0060	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.044	7.5584	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9526	5.4119	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	4.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	4.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.5590	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	4.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1850	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2635	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	55.680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26	7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27	7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28	7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29	7.5976	5.4205	4.5378	4.0449	3.7254	3.4995	3.3302	3.1982	3.0920
30	7.5625	5.3904	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6591	3.3389	3.1187	2,9530	2.8233	2.7185
120	6.8510	4.7865	3.9493	3.4706	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

(continued)

10	12	15	20	24	30	40	60	120	∞
6055.8	6106.3	6157.3	6208.7	6234.6	6260.7	6286.8	6313.0	6339.4	6366.0
99.399	99.415	99.432	99.449	99.458	99.466	99.474	99.483	99.491	99.501
27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
10.051	9.8883	9.7222	9.5527	9.4665	9.3793	9.2912	9.2020	9.1118	9.0204
7.8741	7.7183	7.5590	7.3958	7.3127	7.2285	7.1432	7.0568	6.9690	6.8801
6.6201	6.4691	6.3143	6.1554	6.0743	5.9921	5.9084	5.8236	5.7372	5.6495
5.8143	5.6668	5.5151	5.3591	5.2793	5.1981	5.1156	5.0316	4.9460	4.8588
5.2565	5.1114	4.9621	4.8080	4.7290	4.6486	4.5667	4.4831	4.3978	4.3105
4.8402	4.7059	4.5582	4.4054	4.3269	4.2469	4.1653	4.0619	3.9965	3.9090
4.5393	4.3974	4.2509	4.0990	4.0209	3.9411	3.8596	3.7761	3.6904	3.6025
4.2961	4.1553	4.0096	3.8584	3.7805	3.7008	3.6192	3.5355	3.4494	3.3608
4.1003	3.9603	3.8154	3.6646	3.5868	3.5070	3.4253	3.3413	3.2548	3.1654
3.9394	3.8001	3.6557	3.5052	3.4274	3.3476	3.2656	3.1813	3.0942	3.0040
3.8049	3.6662	3.5222	3.3719	3.2940	3.2141	3.1319	3.0471	2.9595	2.8684
3.6909	3.5527	3.4089	3.2588	3.1808	3.1007	3.0182	2.9330	2.8447	2.7528
3.5931	3.4552	3.3117	3.1615	3.0835	3.0032	2.9205	2.8348	2.7459	2.6530
3.5082	3.3706	3.2273	3.0771	2.9990	2.9185	2.8354	2.7493	2.6597	2.5660
3.4338	3.2965	3.1533	3.0031	2.9249	2.8442	2.7608	2.6742	2.5839	2.4893
3.3682	3.2311	3.0880	2.9377	2.8594	2.7785	2.6947	2.6077	2.5168	2.4212
3.3098	3.1729	3.0299	2.8976	2.8011	2.7200	2.6359	2.5484	2.4568	2.3603
3.2576	3.1209	2.9780	2.8274	2.7488	2.6675	2.5831	2.4951	2.4029	2.3055
3.2106	3.0740	2.9311	2.7805	2.7017	2.6202	2.5355	2.4471	2.3542	2.2559
3.1681	3.0316	2.8887	2.7380	2.6591	2.5773	2.4923	2.4035	2.3099	2.2107
3.1294	2.9931	2.8502	2.6993	2.6203	2.5383	2.4530	2.3637	2.2695	2.1694
3.0941	2.9579	2.8150	2.6640	2.5848	2.5026	2.4170	2.3273	2.2325	2.1315
3.0618	2.9256	2.7827	2.6316	2.5522	2.4699	2.3840	2.2938	2.1984	2.0965
3.0320	2.8959	2.7530	2.6017	2.5223	2.4397	2.3535	2.2529	2.1670	2.0642
3.0045	2.8685	2.7256	2.5742	2.4946	2.4118	2.3253	2.2344	2.1378	2.0342
2.9791	2.8431	2.7002	2.5487	2.4589	2.3860	2.2992	2.2079	2.1107	2.0062
2.8005	2.6649	2.5216	2.3689	2.2880	2.2034	2.1142	2.0194	1.9172	1.8047
2.6318	2.4961	2.3523	2.1978	2.1154	2.0285	1.9360	1.8363	1.7263	1.6006
2.4721	2.3363	2.1915	2.0346	1.9500	1.8600	1.7629	1.6557	1.5330	1.3805
2.3209	2.1848	2.0385	1.8783	1.7908	1.6964	1.5923	1.4730	1.3246	1.0000

Table B-5. F-table $F_{.005}(f_1,f_2)$, 99.5% confidence

f_1 = number of degrees of freedom of numerator
f_2 = number of degrees of freedom of denominator

f_2 \ f_1	1	2	3	4	5	6	7	8	9
1	16211	20000	21615	22500	23056	23437	23715	23925	24091
2	198.50	199.00	199.17	199.25	199.30	199.33	199.36	199.37	199.39
3	55.552	49.799	47.467	46.195	45.392	44.838	44.434	44.126	43.882
4	31.333	26.284	24.259	23.155	22.456	21.975	21.622	21.352	21.139
5	22.785	18.314	16.530	15.556	14.940	14.513	14.200	13.961	13.772
6	18.635	14.544	12.917	12.028	11.464	11.073	10.786	10.566	10.391
7	16.236	12.404	10.882	10.050	9.5221	9.1554	8.8854	8.6781	8.5138
8	14.688	11.042	9.5965	8.8061	8.3018	7.9520	7.6952	7.4960	7.3386
9	13.614	10.107	8.7171	7.9559	7.4711	7.1338	6.8849	6.6933	6.5411
10	12.826	9.4270	8.0807	7.3428	6.8723	6.5446	6.3025	6.1159	5.9676
11	12.226	8.9122	7.6004	6.8809	6.4217	6.1015	5.8648	5.6821	5.5368
12	11.754	8.5096	7.2258	6.5211	6.0711	5.7570	5.5245	5.3451	5.2021
13	11.374	8.1865	6.9257	6.2335	5.7910	5.4819	5.2529	5.0761	4.9351
14	11.060	7.9217	6.6803	5.9984	5.5623	5.2574	5.0313	4.8566	4.7173
15	10.798	7.7008	6.4760	5.8029	5.3721	5.0708	4.8473	4.6743	4.5364
16	10.575	7.5138	6.3034	5.6378	5.2117	4.9134	4.6920	4.5207	4.3838
17	10.384	7.3536	6.1556	5.4967	5.0746	4.7789	4.5594	4.3893	4.2535
18	10.218	7.2148	6.0277	5.3746	4.9560	4.6627	4.4448	4.2759	4.1410
19	10.073	7.0935	5.9161	5.2681	4.8526	4.5614	4.3448	4.1770	4.0428
20	9.9439	6.9865	5.8177	5.1743	4.7616	4.4721	4.2569	4.0900	3.9564
21	9.8295	6.8914	5.7304	5.0911	4.6808	4.3931	4.1789	4.0128	3.8799
22	9.7271	6.8064	5.6524	5.0168	4.6088	4.3225	4.1094	3.9440	3.8116
23	9.6348	6.7300	5.5823	4.9500	4.5441	4.2591	4.0469	3.8822	3.7502
24	9.5513	6.6610	5.5190	4.8898	4.4857	4.2019	3.9905	3.8264	3.6949
25	9.4753	6.5982	5.4615	4.8351	4.4327	4.1500	3.9394	3.7758	3.6447
26	9.4059	6.5409	4.4091	4.7852	4.3844	4.1027	3.8929	3.7297	3.5989
27	9.3423	6.4885	5.3611	4.7396	5.3402	4.0594	3.8501	3.6875	3.5571
28	9.2838	6.4403	5.3170	4.6977	4.2996	4.0197	3.8110	3.6487	3.5186
29	9.2297	6.3958	5.2764	4.6591	4.2622	3.9830	3.7749	3.6130	3.4832
30	9.1797	6.3547	5.2388	4.6233	4.2276	3.9492	3.7416	3.5801	3.4505
40	8.8278	6.0664	4.9759	4.3738	3.9860	3.7129	3.5088	3.3498	3.2220
60	8.4946	5.7950	4.7290	4.1399	3.7600	3.4918	3.2911	3.1344	3.0083
120	8.1790	5.5393	4.4973	3.9207	3.5482	3.2849	3.0874	2.9330	2.8083
∞	7.8794	5.2983	4.2794	3.7151	3.3499	3.0913	2.8968	2.7444	2.6210

(continued)

10	12	15	20	24	30	40	80	120	∞
24224	24426	24630	24836	24940	25044	25148	25253	25359	25465
199.40	199.42	199.43	199.45	199.46	199.47	199.47	199.48	199.49	199.51
43.686	43.387	43.085	42.778	42.622	42.466	42.308	42.149	41.989	41.829
20.967	20.705	20.438	20.167	20.030	19.892	19.752	19.611	19.468	19.325
13.618	13.384	13.146	12.903	12.780	12.656	12.530	12.402	12.274	12.144
10.250	10.034	9.8140	9.5888	9.4741	9.3583	9.2408	9.1219	9.0015	8.8793
8.3803	8.1764	7.9578	7.7540	7.6450	7.5345	7.4225	7.3088	7.1933	7.0760
7.2107	7.0149	6.8143	6.6082	6.5029	6.3961	6.2875	6.1772	6.0649	5.9505
6.4171	6.2274	6.0325	5.8318	5.7292	5.6248	5.5186	5.4104	5.3001	5.1875
5.8467	5.6613	5.4707	5.2740	5.1732	5.0705	5.9659	4.8592	4.7501	4.6385
5.4182	5.2363	5.0489	4.8552	4.7557	4.6543	4.5508	4.4450	4.3367	4.2256
5.0855	4.9063	4.7214	4.5299	4.4315	4.3309	4.2282	4.1229	4.0149	3.9039
4.8199	4.6429	4.4600	4.2703	4.1726	4.0727	3.9704	3.8655	3.7577	3.6465
4.6034	4.4281	4.2468	4.0585	3.9614	3.8619	3.7600	3.6553	3.5473	3.4359
4.4236	4.2498	4.0698	3.8826	3.7859	3.6867	3.5850	3.4803	3.3722	3.2602
4.2719	4.0994	3.9205	3.7342	3.6378	3.5388	3.4372	3.3324	3.2240	3.1115
4.1423	3.9709	3.7929	3.6073	3.5112	3.4124	3.3107	3.2058	3.0971	2.9839
4.0305	3.8599	3.6827	3.4977	3.4017	3.3030	3.2014	3.0962	2.9871	2.8732
3.9329	3.7631	3.5866	3.4020	3.3062	3.2075	3.1058	3.0004	2.8906	2.7762
3.8470	3.6779	3.5020	3.3178	3.2220	3.1234	3.0215	2.9159	2.8058	2.6904
3.7709	3.6024	3.4270	3.2431	3.1474	3.0488	2.9467	2.8408	2.7302	2.6140
3.7030	3.5350	3.3600	3.1764	3.0807	2.9821	2.8799	2.7736	2.6625	2.5455
3.6420	3.4745	3.2999	3.1165	3.0208	2.9221	2.8198	2.7132	2.6016	2.4837
3.5870	3.4199	3.2456	3.0624	2.9967	2.8679	2.7654	2.6585	2.5463	2.4276
3.5370	3.3704	3.1953	3.0133	2.9176	2.8187	2.7160	2.6099	2.4960	2.3765
3.4916	3.3252	3.1515	2.9685	2.8728	2.7738	2.6709	2.5633	2.4501	2.3297
3.4499	3.2839	3.1104	2.9275	2.8318	2.7327	2.6296	2.5217	2.4078	2.2867
3.4117	3.2460	3.0727	2.8899	2.7941	2.6949	2.5916	2.4834	2.3689	2.2469
3.3765	3.2111	3.0379	2.8551	2.7594	2.6601	2.5565	2.4479	2.3330	2.2102
3.3440	3.1787	3.0057	2.8230	2.7272	2.6278	2.5241	2.4151	2.2997	2.1760
3.1167	2.9531	2.7811	2.5984	2.5020	2.4015	2.2958	2.1838	2.0635	1.9318
2.9042	2.7419	2.5705	2.3872	2.2989	2.1874	2.0789	1.9622	1.8341	1.6885
2.7052	2.5439	2.3727	2.1881	2.0890	1.9839	1.8709	1.7459	1.6055	1.4311
2.5188	2.3583	2.1868	1.9998	1.8983	1.7891	1.6691	1.5325	1.3637	1.0000

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